Metaphysical Foundations of Natural Science

Immanuel Kant

1786

[Brackets] enclose editorial explanations. Small ·dots· enclose material that has been added, but can be read as though it were part of the original text. Occasional •bullets, and also indenting of passages that are not quotations, are meant as aids to grasping the structure of a sentence or a thought. Every four-point ellipsis . . . . indicates the omission of a brief passage that seems to present more difficulty than it is worth. Longer omissions are reported on, between [brackets], in normal-sized type. Numerals in the margins refer to the pages in the Akademie edition of the work; these numbers are also supplied in both the existing English translations, which can thus easily be correlated with the present version.

First launched: June 2009

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Preface

467 [Kant launches this work by

distinguishing two senses of the word 'nature'. There is **(n)** its use in phrases of the form 'the nature of. . . ', where a nature is a quality, and **(N)** its use as a proper name, 'Nature', which signifies not a quality but the sum-total of everything that can be an object of our senses and thus also an object of experience. Kant also lingers on **(n)** in order to distinguish 'the nature of. . . ' from 'the essence of. . . , but this won't concern us in the present work, whose central concern is with **(N)** Nature—the whole world that we can know about through our senses. [Because he was writing in German, Kant had to use a capital 'N' for *Natur* in each of those senses. In English we have a choice; and this version will use 'nature' for the 'nature of . . . ' concept and 'Nature' for the name of a single entity.]] Kant continues:] Nature taken in this sense of the word has two main parts, corresponding to the main line through our senses:

> One part contains the objects of the •external senses. Any theory about that will be a doctrine of •body, dealing with extended Nature.
>
> The other part contains the object of the •internal sense. Any theory about it will be a doctrine of •soul, dealing with thinking Nature.

[Kant's *Lehre* has to be translated as 'doctrine', but really that is misleading. He will label as a *Lehre* any disciplined body of fact-and-theory about a given subject; any respectable university department will be dedicated to some *Lehre*; but a *Lehre* can be much too small to support a department—e.g. Kant will speak about 'the doctrine of the properties of a straight line'!] If a doctrine is a *system*—i.e. a knowledge-total ordered according to principles—then it's what we call a *science*. Now, there are two sorts of principles that can
468 connect items of knowledge so that they constitute a *whole*: •empirical principles and •rational principles. This could prompt us to distinguish 'historical natural science' from 'rational natural science'; but it turns out that this is a bad way of stating things. [Kant's explanation of why is confusing: he announces it as focussing on the meaning of 'Nature' but states it in terms of the meaning of 'science'. [We'll see that both items are involved.] The core of the explanation is that *any* natural science, properly so-called, must include principles that rationally hold items of knowledge together. Kant continues:] So the doctrine of Nature—whether extended or thinking—might better be divided into

> **(a)** the historical doctrine of Nature, which contains nothing but the systematically ordered facts about natural things—presenting •Nature as a system of classes of natural things ordered according to similarities, and the •history of Nature as a systematic account of natural things in different times and in different places; and **(b)** natural science.

And natural science *properly* so-called would treat its subject-matter wholly according to *a priori* principles, while natural science *improperly* so-called would treat its subject-matter according to laws of experience.

Nothing counts as science proper unless it is •apodeictically certain, ·i.e. certain because it is absolutely necessary·; any cognitive structure that makes use of merely •empirical certainty is only improperly called 'science'. . . . An example of the latter is chemistry, the basic premises of which are merely empirical; the laws from which the given facts are logically deduced in chemistry are merely laws of experience, which •don't bring with them any consciousness of their necessity and therefore •aren't apodeictically certain. So that entire structure doesn't strictly count as a 'science', and would be better referred to as a systematic art. [This uses 'art', as Kant uses the corresponding word *Kunst*, to mean something like 'disciplined assemblage of skills'.]

So a rational doctrine of Nature deserves the label 'natural science' only when the laws of Nature that underlie it are **(1)** known *a priori* and aren't mere **(2)** laws of experience. Knowledge of Nature of kind **(1)** is called *pure* rational knowledge; knowledge of kind **(2)** is called *applied* rational knowledge. Since the word 'Nature' already carries with it the concept of laws, and since that carries with it the concept
469 of. . . .necessity, it's easy to see •why something can count as natural science only because of the pure part of it, i.e. the part containing the *a priori* principles of all the other explanations of Nature, and •why it's only because of this pure part that it is a *science*. Thus, every discipline dealing with Nature must, according to reason's demands, eventually come to be natural *science*, because the very concept of *Nature* has the necessity of laws inseparably attached to it and required for Nature to be thoroughly understood. [•This removes the confusion mentioned in an earlier note. Kant holds that both the concepts of *Nature* and those of *science* conceptually involve *necessary law*; so any disciplined treatment of Nature must bring in such laws, thereby helping to qualify itself as a science. •Why 'reason's demands'? Because of Kant's doctrine—expounded in his *Critique of Pure Reason* but not here—that reason constantly urges us to interconnect our various items of knowledge, always restlessly trying to get it all into a single rigidly interconnected system.] That is why the most complete explanation of certain phenomena by chemical principles always leaves us dissatisfied, because it has involved only contingent laws learned by mere experience, with no input from anything *a priori*.

Thus all genuine natural science requires a pure part which could be the basis for the apodeictic certainty that reason looks for in such science. And since the principles at work in the pure part make it completely different from the part whose principles are only empirical, there is a lot to be gained from a procedure in which the empirical part is kept out of sight while we expound the pure part on its own, as completely as we possibly can, so as to discover exactly •what reason can accomplish unaided, and •where it starts to need help from principles of experience. . . . ·And now I need to introduce another distinction·:

> •Pure philosophy (= metaphysics) is pure rational knowledge from mere concepts; •Mathematics is pure rational knowledge that is based entirely on the construction of concepts by means of the presentation of the object in *a priori* intuition.

[That account of mathematics comes from a theory of Kant's which is easiest to grasp in application to geometry. Take the proposition that *the total length of any two sides of a triangle is greater than the length of the third side;* how do you know that this is true? Not empirically, by **(1)** measuring the sides of triangular things, or by **(2)** reading it off from the concept *triangle*.

> By method **(1)** we could only get truths known *a posteriori*, i.e. from experience.
> By method **(2)** we could only derive analytic truths—ones knowable through conceptual analysis.

What is remarkable about geometrical truths is that they are known *a priori* and yet are synthetic—i.e. known without appeal to experience but not by being derived purely from concepts. Well, then, how *are* they known? Kant's answer is this: If you *know* that proposition about triangles (and haven't merely taken it on trust from someone else), you must have *constructed a triangle in your mind's eye* and seen from this that the proposition is true. In our present text Kant writes here and below of 'constructing concepts', but that is misleading. He doesn't think that in this process you construct any concept. Rather, you construct, under the guidance of a concept, a mental triangle.] Natural science properly so-called presupposes the metaphysics of Nature, ·i.e. pure rational knowledge from mere concepts·. Why? Because a science properly so-called has to include necessary propositions, and in *this* science they must be necessary truths having to do with the existence of things; so they can't be based on *a priori* intuition, because no such intuition can present anything concerning existence. The necessary propositions involved in natural science, therefore, have to be the concept-based ones that define 'metaphysics of Nature'. There are two possibilities for what they might be:

(1) The metaphysics of Nature might deal with the laws that make possible the concept of *a thing's nature*, without bringing in any specific object of experience, and therefore not saying anything specific about any particular kinds of empirical object. The part of the metaphysics of Nature that
470 does this is its transcendental part. [For Kant a 'transcendental' principle is one that has to do with the conditions that *make possible* some kind of knowledge.]

(2) The metaphysics of Nature might instead deal with the special nature of this or that kind of thing, of which it has an empirical concept; doing this in such a way it doesn't look to experience for anything except this concept. (·If it looked to experience for information, it wouldn't count as 'metaphysics'·.) For example, it takes as its foundation the empirical concept of *a material thing* or the empirical concept of *a thinking thing*, and searches for anything that reason can teach us *a priori* regarding these things. This science would still count as a 'metaphysic' of Nature—specifically, of corporeal or of thinking Nature—but it wouldn't be a •general metaphysic but rather •a special metaphysical natural science (physics and psychology), in which the transcendental principles mentioned in **(1)** are applied to the two sorts of sense-objects.

In any special doctrine of Nature there is only as much genuine *science* as there is mathematics. As I have explained, a science (properly so-called) of Nature must have a pure part that is the foundation for the empirical part and is based upon *a priori* knowledge of natural things. ·Let us now look very carefully into this notion of *a priori* knowledge of natural things·. To know something *a priori* is to know it from its mere possibility. But the possibility of specific natural things ·such as bodies and minds· can't be discovered from their mere concepts. ·For example·: from the concept of *body* we can discover the possibility of having a self-consistent thought about a body, but we can't discover the possibility of a body as a natural thing that could exist outside of the thought of it. So if we are to have knowledge of the possibility of specific kinds of natural things, and hence to know ·truths about· them *a priori*, we'll need to be given *a priori* an intuition corresponding to the concept, i.e. we need the concept to be constructed. *And rational knowledge through the construction of concepts is mathematical.* It may be possible to dispense with mathematics in developing a •pure philosophy of Nature in general, i.e. one whose only topic is what constitutes the concept of a nature in general; but a pure doctrine of Nature concerning specific natural things (a doctrine of body or a doctrine of soul) is possible only through mathematics. . . .

[That's why chemistry can't be a *science*, Kant says. For
it to be a science it would have to derive chemical laws about 471
how different sorts of matter react with one another from an *a priori* intuition—something constructed in our minds. And there is no chance of that. And so, Kant continues,] chemistry can't be anything more than a systematic art or experimental doctrine, never a science proper, because the principles of chemistry are merely empirical and can't be presented *a priori* in intuition. . . .

But the empirical study of the soul must always be even further from qualifying as a natural *science* than chemistry is. Why? Because mathematics can't be applied to the phenomena of inner sense and their laws. ('But the flow of inner sense's internal changes is *continuous*, and continuity can be treated mathematically.' Yes, but •what *that* could add to the content of the doctrine of the soul is vastly less than •what mathematics can add to the content of the doctrine of body; in about the way that •the doctrine of the properties of a straight line are less than •the whole of geometry! In each case, the tiny doctrine concerns only a single dimension—in the case of the soul it's the single dimension of *time*.) Anyway, if we ·keep mathematics out of the picture and· think of the doctrine of the soul merely as as a •systematic art of analysis or as an •experimental doctrine, it still falls wells short of chemistry, ·in three ways·. **(i)** Given any two elements in the complex of events observed through inner sense, I can *think of them them separately*, but I can't *separate them* and then bring them together as I choose. **(ii)** I can't investigate the mental events in someone else's mind. **(iii)** With mental events, ·unlike chemical ones·, an observed event can be altered and distorted by the mere fact of being observed. So the doctrine of the soul can't be anything more than . . . . a •natural description of the soul, not a •science of it, and not even a •psychological experimental doctrine. That is why in the title of this work—which really contains only the principles of the doctrine of *body*—I have followed standard usage in employing the general name 'natural science'; for strictly speaking it's only the doctrine of body that is entitled to be called 'science'.

472 But it can't be *natural* science unless mathematics is brought into it, and that can't happen until. . . .a complete analysis of the absolutely general concept of *matter* has been provided. Providing that is the business of pure philosophy. That general concept is an empirical one, but pure philosophy ·in dealing with it· doesn't make use of any particular experiences; it employs only what it finds in the concept of matter that relates to pure space and time. (Such relations come from laws that depend essentially on the concept of *Nature*.) Such a doctrine of body is, therefore, an actual metaphysics of corporeal Nature.

So all natural philosophers who have wanted to proceed mathematically in their work have availed themselves (without realizing it) of metaphysical principles; they *had* to do so, despite their solemn declarations that metaphysics has no claims on their science. No doubt they took 'metaphysics' to be a light-minded activity of •inventing possibilities at will and •playing with concepts which might be incapable of being presented in intuition and have as their only claim to objective reality the mere fact that they aren't self-contradictory! All true metaphysics comes from the essential nature of our thinking faculty, so it's not something we *invent*. The content of metaphysics doesn't come from experience; ·it's nearer the truth to say that experience comes from metaphysics!·. Metaphysics consists in the pure operations of thought—*a priori* concepts and principles whose basic role is to bring the elements of the tangle of empirical representations into lawful connection with one another, thereby turning the tangle into *experience*. That's why those mathematical physicists couldn't do without some metaphysical principles, including the ones that make the concept of their own special object—*matter*—available *a priori* for application to external experience, as with the concepts of motion, of the filling of space, of inertia, and so on. But they rightly held that the apodeictic certainty they wanted their natural laws to have couldn't be had by any merely empirical principles; so they preferred to *postulate* such laws without investigating their *a priori* sources.

In the pure part of natural science as ordinarily conducted, metaphysical and mathematical constructions crisscross with one another; ·and that is very unsatisfactory·. It is enormously beneficial for the sciences to keep principles of different kinds at a distance from each other, putting •each 473
kind into a separate system which constitutes a science

of •that kind. If this isn't done, people can confuse them with one another, failing to see which kind is relevant to a particular problem. . . . That is why I have thought it necessary to segregate •metaphysical principles from the pure part of natural science that has usually been the stamping-ground of metaphysical as well as mathematical constructions, putting •them into a system of their own—a system that will also contain the principles of the construction of those ·mathematical· concepts, and therefore the principles of the possibility of a mathematical doctrine of Nature itself. ·But this system won't contain any mathematics·. . . .

Here is a second advantage of this procedure. In anything that is called 'metaphysics' we can hope for absolute completeness, which can't be expected in any other branch of knowledge; and we can confidently expect such completeness not only for the metaphysics of Nature in general but also for our present topic of the metaphysics of corporeal Nature. Why can we expect this? Because in metaphysics the object—·the item you are studying·—is considered merely as it has to be represented in accordance with the universal necessary laws of thought; this confines the possible results to a definite number of items of knowledge, and it's possible to come to have *all* of these. In contrast with this, in any other science we consider the object as it has to be represented in accordance with data of intuition; there is a limitless web of intuitions, and therefore of objects of thought, so that the science can never achieve absolute completeness, but can be endlessly extended, as in pure mathematics and the empirical doctrine of Nature. [In that sentence, Kant twice specifies that the intuitions he is talking about include pure as well as empirical ones.] I think that ·in the present work· I completely exhaust the metaphysical doctrine of body, extend it as far as you like; but I don't regard that as much of an achievement.

The schema for the completeness of a metaphysical system, whether of Nature in general or of corporeal Nature in particular, is the table of the categories.[1] That is because this 474

[1] [In an enormous footnote Kant reports that something published in a recent issue of one of the learned journals expresses doubts relating to his use of his 'table of the pure concepts of the understanding'. He continues:] The doubts aren't aimed at the table itself, but at the conclusions I have drawn from it regarding the limitations of the whole faculty of pure reason and therefore of all metaphysics. . . . These doubts are supposed to touch the main foundation of my system, as set out in the *Critique of Pure Reason*. . . . This main foundation is said ·by my critic· to be my deduction of the pure concepts of the understanding, expounded partly in the *Critique* and partly in the *Prolegomena*. That part of the *Critique* (·says my critic·) should have been the clearest but is actually the most obscure or indeed argues in a circle, and so on. The chief point in these objections is the claim that without a completely clear and adequate deduction of the categories, the system of the *Critique of Pure Reason*, far from being apodeictically certain, would totter on its foundation; and that is what I shall answer here. [Kant's answer is long, dense, difficult, and not needed for present purposes. The gist of it involves his taking his critic to agree •that the categories are forms of thought that we have to use in intellectually dealing with whatever we have to think about, and •that all we can ever have to think about are appearances. These concessions, Kant says, give him his core thesis in the*Critique*, namely that the categories represent the limits to what thoughts we can have, what propositions we can entertain, and so on; and he represents his critic as accepting *that* the categories do this while complaining that Kant hasn't explained *how* they can do it. He replies that his system doesn't need the*how*, which is mere icing on the cake [not his formulation!]. He says that if his account of *how* were a failure, he would still be in good company:] Newton's system of universal gravitation is well established, despite our continuing difficulty about explaining *how* attraction at a distance is possible. Difficulties are not doubts. [And then Kant re-states all this at much greater length, ending up with a slap at his critic, saying that when certain things are made clearer in the second edition of the *Critique*,] that will spare my critic from having to resort to a pre-established harmony because of the surprising agreement of appearances with the laws of the understanding. This 'remedy' is much worse than the

table contains *all* the pure concepts of the understanding that have something to do with the nature of things. It
475 must be possible to bring under the four kinds of category—quantity, quality, relation, and modality—all detailed
476 special cases of the universal concept of *matter*, and therefore everything that

can be •thought *a priori* concerning matter, •presented in mathematical constructions, or •given in experience as a determinate object of experience.

There's nothing more to be discovered or added; but there may be room for improvements in clearness or thoroughness.

Accordingly, the present work contains four chapters, each dealing with matter brought under one of the four kinds of concepts of the understanding. Something that is present in all the chapters is *motion*. The senses can't be affected by matter unless something moves; so motion is the *basic* fact about anything that is to be an object of the external senses; and the understanding leads all other
477 predicates that express the nature of matter back to motion; so natural science is, *throughout*, either a pure or an applied doctrine of motion. The *Metaphysical Foundations of Natural Science* can therefore be divided into four chapters.

1. Phoronomy: In this, motion is considered as pure quantum—portions of which can be combined in various ways—with no attention being paid to any quality of the matter that moves. [See note on page 18]

2. Dynamics: This treats motion as belonging to the quality of the matter under the label 'basic moving force'. [See note on page 39]

3. Mechanics: This deals with how the movements of portions of matter bring them into ·causal· relations with one another. [See page 58]

4. Phenomenology: In this chapter, matter's motion or rest is handled purely in terms of how it is represented—its modality—and thus in terms of its status as an appearance of the external senses. [See note on page 62]

I have shown the necessity of distinguishing •the metaphysical foundations of the doctrine of body not only from •physics (which employs empirical principles) but even from •physics's rational premises, which concern the employment of mathematics in physics. The reasons for that were internal to metaphysics; but there's also an external reason to deal thoroughly with the doctrine of body as a separate unit, not mixing it up with the general system of metaphysics. This external reason is only accidental—·it depends on a sheer fact about how certain people behave·—but it is important. We can mark the boundaries of a science not merely in terms of •its subject-matter and of •the specific kind of knowledge of that subject-matter, but also in terms of •what those who pursue the science have in mind as a *use* for it. Well, what do all the people who have busied their heads with metaphysics—and will continue to do so —had in mind as a use for it? They have planned for it to

•extend natural knowledge

(which they could do much more easily and certainly by

evil it is meant to cure. . . . Such a pre-established harmony can't generate the objective *necessity* that characterizes the principles in which pure concepts of the understanding are applied to appearances. For example, it provides no basis for cause-effect connections to be objectively necessary (though it allows subjective necessity (·when we experience C we *can't help expecting* E to follow·). The only possible basis for this objective necessity is the *a priori* principles that lie at the foundation of the possibility of thought itself, these being needed if we are to have knowledge of objects whose appearance is given us, i.e. if we are to be able to have *experience*. And even if there could be no adequate explanation of *how* experience is possible in the first place, it would still be indisputably certain that experience is possible only through those concepts and, conversely, that the only meaningful use for those concepts is in relation to objects of experience.

observation, experiment, and the application of mathematics to external phenomena), and also to

•give them knowledge of what lies entirely beyond all the boundaries of experience, namely God, freedom, and immortality.

These things being so, there is a lot to be gained by handling the metaphysics of the doctrine of body in isolation from the rest of metaphysics ·rather than letting it get caught up in that jumble of concerns·. It does in fact grow from general metaphysics, and that shouldn't be forgotten; but it will grow better if we treat it as having been planted in its own ground. This won't affect the completeness of the system of •general metaphysics. It will indeed make it easier for •this science to progress smoothly towards its goal if,
478 whenever it needs to bring in the general doctrine of body, it can call upon the separate system of such a doctrine without having had to include it in its baggage all along. And there's another significant fact (which I can't go into in detail here), namely that general metaphysics, whenever it needs to provide examples (intuitions) to give meaning to its pure concepts of the understanding, always has to take them from the general doctrine of body, i.e. from the form and principles of *external* intuition. And when such examples are not ready at hand, general metaphysics gropes, shaking with uncertainty, among mere meaningless concepts. . . . So a separate metaphysics of corporeal Nature does excellent and indispensable service to general metaphysics . . . . In the present work I have modelled my procedure on the mathematical method —not making my work strictly mathematical (I hadn't time for that), but treating mathematics as something to imitate. This isn't meant as a display of profundity that might earn the work a better reception. Rather, it reflects my belief that a system such as this is quite capable of a mathematical treatment, and that it may some day be completed by someone cleverer than I am. That could happen when mathematical investigators of Nature, stimulated by this sketch of mine, think it worthwhile to extend their studies to the metaphysical portion ·of the doctrine of body· . . . . and to bring it into unison with the mathematical doctrine of motion.

In the preface of his *Principia*, Newton follows up his remark that geometry needs to postulate only two mechanical actions, the ones that trace a straight line and a circle, by saying: 'Geometry is prioud of being able to produce so much, with so little taken from elsewhere.' In contrast with that, one might say of metaphysics: It stands astonished that with so much offered to it by pure mathematics, it can achieve so 479
little! Nevertheless, this 'little' is something that mathematics absolutely *has* to have in its application to natural science; and since mathematics must here necessarily borrow from metaphysics, it shouldn't be ashamed to be seen in the company of the latter. [From here on, displayed occurrences of 'Definition' translate Kant's *Erklärung*, which usually means 'explanation'. Kant hinself licenses this somewhat loose use of 'definition' in his *Critique of Pure Reason* B 75.]

Chapter 1
Metaphysical Foundations of Phoronomy

Definition 1

I call something '**material**' if and only if it is movable in 480
space. Any space that is movable is what we call 'material' or '**relative**' space. What we think of as the space in which all motion occurs—space that is therefore absolutely immovable—is called 'pure' space or '**absolute**' space.

Remark 1

The *whole* topic of phoronomy is motion; so the *only* property that is here attributed to the subject of motion, i.e. matter, is its *movability*. So we are free to take ·any portion of· matter as a *point*. In phoronomy we set aside all the internal characteristics of matter, thereby setting aside anything involving the quantitative notion of *how much* matter we are dealing with; all we are concerned with is the motion of matter, and the only quantitative notion that we need is not *how much matter* but only *how fast* and *in what direction* the matter moves. [Why does Kant imply that direction-of-movement is *quantitative*? Because he is thinking of 180-degree changes of direction of straight-line movements; an N movement in one direction can be thought of as a minus-N movement in the opposite direction.] If I sometimes use the expression 'body'—·meaning *a body*, not merely undifferentiated *matter*·—that will be because I am deliberately getting ahead of myself, making my discourse less abstract and more comprehensible by bringing into phoronomy some of the more determinate concepts of matter that we shall come to later.

Remark 2

481 If I explain the concept of *matter* not by a predicate that applies to it as object—

> ·i.e. not by saying anything of the form 'any item is matter if it has property P·'

—but only by how it relates to the knowledge-faculty through which it is basically represented to me—

> ·i.e. by saying 'matter is whatever is represented to me by outer sense'·

—then 'matter' is being explained as applying to every object of the external senses; and this would be the mere metaphysical definition of it. But space would be simply the form of all external sensible intuition. . . . [That last phrase refers to the use of our senses in application to the external world; it stands in contrast with *a priori* intuition—see the long note on page 2.] In contrast to this *form*, matter would be what our outer senses give us sensations *of*; so it would be the properly empirical part of external sensory intuition, because matter cannot be given at all *a priori*. In all experience •something must be sensed, and •this is the *real* component in sensible intuition. So the space in which we are to set up experience concerning motions must also be perceptible, i.e. must be indicated by what is perceptible; and this space—

> the sum-total of all objects of experience, and itself an object of experience

—is called 'empirical space'. Now, if such a space is material, it is itself movable. But a movable space, if its motion is to be perceptible, presupposes a larger material space for it to move in, this enlarged space presupposes one larger still, and so on to infinity.

Thus, all motion that is an object of experience is merely relative. We have

> an object x which we perceive to move;
> a space S_1 relative to which we perceive x to move;
> a larger space S_2 relative to which S_1 may move.

It might happen that S_1 does move relative to S_2, and indeed moves in the opposite direction to x ·and at the same speed·; in which case we can describe x as 'moving' in relation to S_1 and at the same time 'motionless' with respect to S_2. These varying accounts of whether and how x moves continue infinitely as we bring in larger and larger relative spaces. [Kant now has a long sentence that is hideously unclear, apparently because it is too compressed. The gist of it seems to be as follows. An absolute space —i.e. a space that isn't material 482
because it isn't movable—is something we assume because it is required for the possibility of experience. But in doing this we are assuming something that can't be perceived •in itself or •in its consequences. (Perceiving it in its consequences

would be perceiving something that we knew was a case of some object moving relative to absolute space, and there's no way we can perceive that.) Furthermore, although we need this assumption for the possibility of experience, we never have any experience in which absolute space plays a part. The whole story of what we perceive can't give any role to absolute space. Kant continues:] So absolute space is in itself *nothing*; it's not any kind of *object*. All it signifies is this: Whenever I am thinking about some object that is moving relative to some space S—·e.g. a leaf blowing through the window and falling onto the carpet in my study·—my thought of 'absolute space' AS is just my thought of every other relative space that I can think of as containing S, the series of such ever-larger spaces running to infinity. This is just a *thought* that I have; I'm not confronted by anything—any matter—that *indicates* this space AS; so my thought represents AS as pure, nonempirical, and absolute. I can compare any empirical space S with AS, representing S as movable in AS, which is therefore always taken to be immovable. If you regard AS as an actual *thing*, you have mistaken

•the *logical* universality that consists in our ability to regard *any* empirical space as being included in it, for
•a *physical* universality that consists in its actually containing *every* empirical space. . . .

[Kant speaks of this mistake as a case of 'misunderstanding reason in its idea', using 'idea' (German *Idee*) as a technical term that he introduced in the *Critique of Pure Reason* and employs just seven times in the present work. (This version will use 'idea' only in translating *Idee*.) For a grasp of how it works, you need to start with the **understanding** and the **concepts** that are its tools. We can have a concept of x only if we could be 'given' an example of x in experience; so we have a concept of *division of a bit of matter* because we can see or feel a bit of matter being cut into two or four or. . . What about •the thought of *division of a bit of matter carried the whole way*? Unless you think that there are 'atoms', smallest bits of matter that can't be further divided, •this thought goes with the thought of an infinitely small bit of matter; that is something we couldn't conceivably encounter in experience; so we have no concept of it; but we do have the idea, this being a thought that takes some concept and subjects it to the thought of *going the whole way* or (in terminology that Kant uses a lot in the *Critique* but not in the present work) the thought of a certain kind of *totality*. It is the role of reason, he holds, to engage in this totalising sort of thought, which is why he links ideas with reason, as he links concepts with understanding. In the use of 'idea' that we have just encountered, Kant speaks of the totalising activity as involving a 'logical universality', and he is referring to the totalising that is involved in the thought of *the whole of space*.] One last remark: An object's movability in space can't be known *a priori*, i.e. without instruction from experience; which is why in the *Critique of Pure Reason* I couldn't count such movability as one of the *pure* concepts of the understanding. The concept of movability, just because it is empirical, can find a place in a natural science only as a bit of *applied* metaphysics, which is where concepts given through experience are dealt with, though according to *a priori* principles.

Definition 2

The **motion** of a thing is the change of its external relations to a given space.

Remark 1

I have based the concept of •matter on the concept of •motion. That's because I wanted to fix the concept of matter without bringing in the concept of •extension, so that I could consider matter as a point, helping myself to the common definition of *motion* as *change of place*. But if we are to define the concept of matter in a comprehensive way that covers moving bodies, that 'change-of- place' definition won't do. The place of any

body is a point. The distance of the moon from the earth is given by the shortest line between their places, i.e. betwen their central points. (That's the only way to get a determinate single distance between them; any other approach will have us measuring from some arbitrarily chosen pair of points—·say the distance from the lowest point in the Dead Sea to the highest point in the Mare Frigoris·.) Now, taking a body's place to be its central point, *a body can move without changing its place*, as the earth does by turning on its axis. But although the rotating earth doesn't change its place, it does change its relation to external space, because at different times it turns different sides toward the moon, and these differences produce all kinds of different effects on the earth. The equation of 'motion' with 'change of place' holds only for movable *points*, i.e. physical points. [The next bit is awkwardly written, but its content can be made clear. Its point is just that the change-of-place definition omits more things than just rotation; it omits, for example,
483 the movements that go on when beer is fermenting in a cask. What the definition applies to is movement of the cask-and-contents as a unit—movement *of* the cask, not movement *in* the cask.] . . . .

Remark 2

Motions can be divided into two classes. **(1)** Progressive movements, which enlarge their space; straight-line movements and curved-line movements that don't return in on themselves. **(2)** Rotatory movements, which don't enlarge their space, but keep returning in on themselves, staying with the same limited space. And these can be divded in turn, into **(2a)** circular movements like those of the planets around the sun and **(2b)** oscillatory movements like that of a pendulum. . . . I mention these different kinds of motion in phoronomy merely because the word 'speed' is generally used in one sense for movements in class **(1)** and a different sense for movements in class **(2)**, as you will see in a moment.

Remark 3

In any motion we have just two factors to think about—*speed* and *direction*—once we have set aside all the other properties of the moving thing. I am here taking for granted the usual definitions of both of these, but various limitations have to be built into the definition of *direction*. . . .

Consider two snails that are exactly alike in shape and even size, except that one winds to the right and the other to the left. What does this difference rest on? Or the difference between the winding of beans around their pole (like a corkscrew—'against the sun', as sailors would say) and the winding of hops, which go around their pole with 484
the sun? We have here an *internal* difference between the two snails, or between the pole-climbing plants—·it's 'internal' in the sense that we can't make it disapear by re-arranging *other* things in certain ways·. Now, the concept of this internal difference can be *constructed*, but it can't be expressed in general terms. It can happen that two things differ *only* in this way, i.e. without this difference bringing others in its train. Take the rare case of a human being who is found through an autopsy to have all his organs inter-related according to the physiological rules that hold for other human beings except that they are left/right reversed. This can't possibly have made any difference to the internal workings of that person's body. And yet there is a real mathematical and indeed internal difference between two motions that differ only in that way, e.g. two circular motions differing in direction but exactly alike in all other respects. [Kant adds his claim that this left/right matter confirms his view that 'space in general doesn't belong to the properties or relations of things in themselves' but 'belongs merely to

the subjective form of our sensible intuition'. He remarks that he has dealt with this elsewhere [in *Prolegomena* section 13]. He continues:] But this is a digression from our present business, in which we *have to* treat space as a property of the things we are considering, namely bodies, because bodies themselves are only phenomena of the external senses and need to be explained here only as such.—So much for *direction*. As for *speed*: the meaning of this expression also varies in different contexts. We say that the earth rotates on its axis 'faster' than the sun because it completes a rotation in a shorter time, although the motion of the earth in this rotation is *slower* than that of the sun. [Kant gives other examples, without suggesting that this point matters much for his present work. He concludes:] In phoronomy we use the word 'speed' with a merely spatial meaning—the measure of how far a thing travels in a given period of time.

Definition 3

485 **Rest** is time-taking presence in the same place; for something to be **time-taking** is for it to exist throughout a time. [The translation makes this look trivial, but it doesn't in the German.]

Remark

A moving body is momentarily at each point of the line that it traverses. Is it at rest at each point or is it moving? No doubt you'll want to say that it is moving, because it is precisely by *moving* that it came to be at this point. But let's consider what is going on in a movement ·I'll call **Oscillate**·, in which

> a body tracks the line AB, from A to B and then back to A again, doing this with a uniform speed so that the total time is exactly one second—half a second from A to B and half a second for the return journey.

This can't happen unless the body doesn't spend any time—not the smallest portion—at B. Why? Because it is present at B only once in Oscillate; allow its presence there to occupy a tiny period of time and you'll have the problem of which of the two journeys—AB or BA—to assign it to. Either way, the times for the two sub-journeys won't be equal. Now change the example to a movement (·I'll call it **Straight**·) in which

> a body moves exactly as in Oscillate except that instead of switching back at B it continues straight on to a further point C.

In Straight the body is moving at B, not at rest. (Why? Because B is just one point in a continuously moving journey, with nothing special about it except that we have chosen to talk about it. If the body weren't moving at B it wouldn't be moving at any point along the A–C line, which means Straight didn't occur.) But Straight is supposed to be exactly like Oscillate except for the directional difference; so if the body is moving at B in Straight then it is moving at B in Oscillate too—but we have just shown that it can't be! Now consider a third example, of a movement ·that I'll call **Updown**·, in which

> a body rises from A *up* to B which is directly above A, and then—having lost its motion by means of gravity when it reaches B—it falls back again from B to A.

In *this* case is the body moving at B or at rest there? The most plausible answer is this:

> In Updown the body is at rest at point B; because when it is there it has been deprived by gravity of all its upward motion, and the downward motion that gravity will also give to it hasn't yet begun. And something that doesn't have any motion is at rest.

But if that is all right for Updown, why isn't it also all right for Oscillate; for in the latter also the return journey from 486
B to A can't start until the forward journey from A to B has ended; so that in Oscillate also we seem to have to conclude that the body is not, after all, moving at B. But we *can't* draw

that conclusion, because something that is moving with a uniform speed can't be at rest anywhere along its journey. What, then, makes the crucial difference between Oscillate and Updown? It is that in Updown the body's motion isn't uniform—it is uniformly decelerated and then uniformly accelerated, in such a way that its speed at B is reduced not to *nothing* but only to *a speed S that is smaller than any assignable speed*. Keep gravity out of this for a moment, and suppose that the body with speed S in Updown doesn't start to fall at B but keeps moving upwards. ·How far would it get, in how much time, if it stayed at speed S? The answer is this·:

> Take any distance you like, however small, along the line up from B, the body wouldn't cover *that* distance, however long it kept moving with speed S.

This implies that (for any possible experience) the body would remain at B for all eternity. Consequently, it is put into a state of •time-taking presence in the same place, i.e. a state of •rest, although owing to the continuous influence of gravity, i.e. the change of this state, the rest is immediately abolished. To •be in a time-taking state is conceptually different from •spending time in that state. . . . Thus *rest* can't be defined as *lack of motion*, because that is negative and so can't be constructed. It must instead be defined as *time-taking presence in the same place*. This can be constructed, by representing a motion with *infinitely small speed* through a period of time that is not infinitely short; and because it can be constructed it can be used in applying mathematics to natural science.

Definition 4

To **construct** the concept of a *composite motion* means to present *a priori* in intuition a motion as the result of two or more given motions united in one movable thing.

Remark

In constructing a concept one mustn't make use of any input from experience, e.g. presupposing some force that one knows about only from experience. Putting the point in 487
its most general form: in constructing a concept one musn't use any concept that can't be given *a priori* in intuition—such as the concepts of cause and effect, and of action and resistance, etc. Don't lose sight of the fact that phoronomy's *only* concern is with the construction of motions in general as *amounts*, so that it takes matter merely as *something movable*, ignoring any facts about *how much* matter is moving in any given case. So phoronomy has from the outset to characterize these motions solely as amounts determined by their speed, their direction and their composition. That much has to be settled entirely *a priori* and indeed through intuition, setting things up for applied mathematics. For the rules governing how motions are inter-connected through physical causes—i.e. forces—can't be properly explained until there's a mathematically constructed basis containing the principles of their composition in general.

Principle

Every motion that could be an object of experience can be viewed either as •the motion of a body in a space that is at rest or as •the rest of a body in a space that is moving in the opposite direction with equal speed. It's a free choice.

Remark

We can experience the motion of a body only if both •the body and •the space in which it moves are objects of external experience—hence, only if they are both material. [Remember that Kant has said that he calls a space 'material' if it can move relative to a larger space.] So an **absolute** motion—i.e. a motion related

to an immaterial space—can't possibly be experienced and is hence nothing at all *for us* (even if we allow that absolute space is something *in itself*). But in all **relative** motion the space itself, because it is assumed to be material, can be represented as at rest or as moving. I represent the space as at rest when it isn't included in some larger space in relation to which I could see it as moving. And I represent the space as moving when it *is* included in some such larger space; an example would be seeing a ball roll along a table in the cabin of a ship, where there is a larger space (including the shore) beyond the space of the cabin, in relation to which •the cabin's space is moving and—it may happen—
488 •the ball is at rest. ·But then the shore's space may be enclosed in a •still larger space relative to which the •shore's space is moving and the •cabin's space is at rest and the •ball is moving after all·! With respect to *any* empirically given space, we can't rule out its being enclosed in a still larger space in relation to which it may be moving or not moving. Thus, for all experience and for every inference from experience, it can't make any difference whether I choose to •consider a body as moving or rather to •consider the body as at rest and the space it is in as moving in the opposite direction with the same speed. The two ways of looking at it are strictly equivalent. You might think that in relation to absolute space one of the accounts is right and the other wrong, but absolute space can't possibly enter into any experience of ours, so we can set it aside. The only difference between body-moving-in-motionless-space and space-moving-around-motionless-body is in how we connect them with other phenomena in our theories.

Also, our experience can't enable us to pick out a fixed point by reference to which we could give sense to a distinction between •absolute motion and •absolute rest. Why not? Because everything we confront in experience is •material, and therefore •movable, and therefore •perhaps actually moving without our being able to perceive this motion. . . . When a body moves in empirical space, I can think of any proportion of the given speed—from *none* to *all*—as belonging to the body, and the remainder—from *all* to *none*—as belonging to the space moving in the opposite direction. There can't be any empirical evidence that would favour any particular distribution. In saying this I am assuming that we are dealing only with motion in a straight line. When other motions are concerned, there isn't the same freedom of choice about what to attribute to the body and what to the space. For example, as between

> •the earth rotates daily on its axis, while the surrounding space (the starry heavens) stay at rest

and

> •the earth remains still while the starry heavens revolve around it,

there are empirically detectable differences. I shall discuss this later on [starting on page 61]. In phoronomy, then, where I consider the motion of a body only in relation to space (upon whose motion or rest the body has no influence at all), it is an arbitrary matter how much (if any) of the speed of a given motion I attribute to the body in question and how much (if any) I attribute to the space that contains it. Later on, ·in mechanics·, where we'll consider how a moving body interacts causally with other bodies in the space of its motion, it will make a discoverable difference how we distribute the speed between the moving body and the space containing it. I'll show this in the proper place [starting at page 53].

Definition 5

The **composition of motion** is the representation of the 489 motion of a point as identical with two or more motions of the point combined.

Remark

Since in phoronomy I don't have thoughts of any quality of matter other than its movability, I can consider matter itself only as a mere point, and can consider any motion as a track through a space. But that doesn't mean that I am attending only to the space that geometry deals with; because I also bring in the •time involved and hence the •speed of the point's movement through space. So phoronomy is the pure doctrine of the amounts of motions.

what comes next, conservatively translated: The determinate concept of a an amount is the concept of the production of the representation of an object through the composition of the geneous.

what Kant seems to have meant: Any *how-much* thought is the thought of a process of mentally assembling something out of parts that are all of the same kind as it. In thinking about (say) *a gallon of water* one is somehow thinking of mentally building up a gallon drop by drop.

Now, nothing is homogeneous with motion except motion, so phoronomy is a doctrine of

> the putting together of different motions of a single point according to their direction and speed,

which is the same as

> the representation of a single motion as comprising within itself two or more motions occurring at the same time.

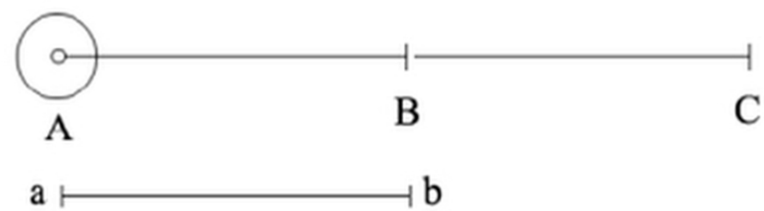
Note that this concerns two or motions that *constitute* one motion; ·all there is to the one motion is those two or more put together·; we are *not* concerned here with two or more motions that *cause* some single motion to occur. In order to find the motion arising from the composition of several motions—as many as you want—you have to proceed piece-meal (as we do with the production of all quantities): start by working out the motion that comes from compounding two of the motions, then compound this with a third. . . and so on. So the doctrine of the composition of *all* motions comes down to the composition of *two*. [Kant goes on to say that there are three different ways in which two motions—whether of equal or unequal speeds—can be happening in a single point at the same time: They may be going **(1)** in a straight line in the same direction, **(2)** in a straight line in opposite directions, or 490
(3) along different lines that are at an angle to one another.]

Proposition

The only way to think of two motions as composing the motion of a single point is by representing •one of the two as occurring in absolute space, and •the other as consisting in the movement of a relative space in the opposite direction.

Proof

First case: A single point undergoes two motions in the same direction along the same line at one time.



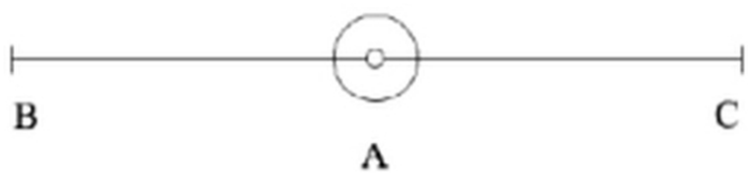
[Kant's presentation of this part of his proof is very hard to follow. It starts with this line of thought: In phoronomy we can equate a speed with a distance/time pair, as we do when we name a speed in terms of 'miles per hour'. Now, suppose that a point is subject to two movements at once, both in a straight line and in the same direction, and think about how

we can represent their speeds. If they are equal, then their speeds can be represented by the AB and ab lines in Figure 1. But. . . The preparer of this version of the text is defeated by what comes next. We are threatened with some kind of incoherence or contradiction in representing the speeds of
491 the two movements on the assumption that they are both movements of a single point x relative to a single space.

[The difficulty is solved, Kant tells us, if we take one of the movements to be a left-to-right movement by x from A to B and take the other to be a right-to-left movement of some relative space that also contains x.

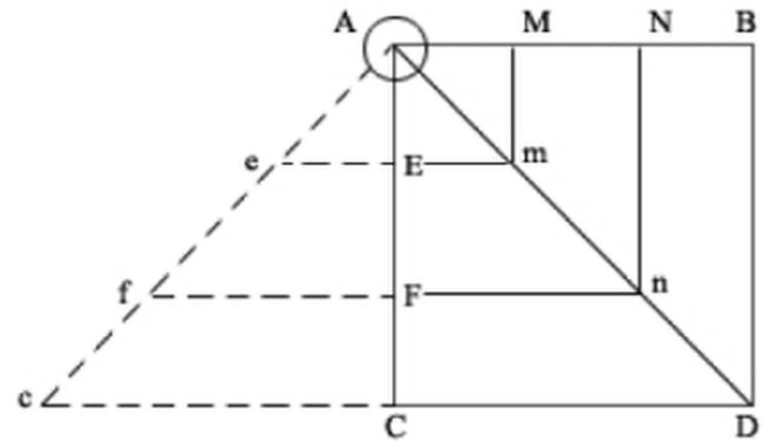
[That is straightforward enough, but the difficulty it is supposed to remedy defeats understanding. The passage in question is presented, closely following the two currently available translations of this work (which differ very little in their handling of this passage) on page 66.]

Second case: Two motions in exactly opposite directions are to be combined at one and the same point.



Let AB be one of these motions and AC the other in the opposite direction (and again let's take the speeds to be equal). In this case the very thought of representing two such motions of a single point x in relation to single space at the same time is plainly impossible. If we are to make sense of the notion of two equal and opposite motions of a single point at the same time, we'll have to think of it as involving a point x that moves in a certain direction relative to absolute space while the relative space that also contains x is moving in the same direction at the same speed. The upshot of this, of course, that relative to the relative space x doesn't move at all. [When in contexts like this Kant speaks of 'the relative space' that is involved, we can take him to mean something like 'the smallest intuitively convenient space' that is involved, out of the possibly infinite series of ever-larger relative spaces that x is contained in.]

Third case: Two motions of a single point go in different 492
directions—not *opposite* directions but *different* ones that enclose an angle.



·To start with, ignore the dotted lines and attend to the square.· Let the two motions we are concerned with be AB and AC. (The angle BAC could be any non-acute angle; it doesn't have to be a right angle as it is here.) Now, if these two motions occur at the same time in the same space, they will go in the *directions* AB and AC but they won't follow the *lines* AB and AC, but only lines parallel to these. The moving point will go through m; and this will be as though the AB movement had pulled the AC movement over to the line Mm, and the AC movement had pulled the AB movement down to the line Em. [Of course all of that should be said first about •a point between A and m, and before that about a point between A and •that point, and. . . and so on. To draw this properly we would need infinitely many smaller squares within the big one! But jumping across and down to m is sufficient for Kant to make his case.] If the directions are to remain the same, therefore, one of the two motions must be altering the other. [Actually, *each* motion must alter the other.] But the Proposition we are proving is about what the two motions *compose*; and the meaning of that (see Definition 5 on page 14) is that the two will jointly *be* the motion in question, not that by changing one another they'll *produce it.*

On the hand, that our moving point x undergoes motion AC in absolute space while—instead of x's undergoing motion AB—some relative space that x is in undergoes motion BA. Then while x moves AE in absolute space, the relative space moves Ee, that is, moves to the left, so that x's position in the relative space is m. And the same story holds for x's
493 absolute move AF while the relative space moves Ff; and for x's entire absolute move AC while the relative space moves Cc. From the standpoint of the relative space, therefore, x moves smoothly down the diagonal, through m and n ·and of course all the intermediate positions· to D, which is exactly the same result as if it had undergone movements AB and AC. So we get the result we want without having to postulate two motions that affect one another.

Remark 1

•Geometrical construction requires that two amounts when put together **are** a third amount, *not* that they **produce** the third in a causal way—for that would be •mechanical construction. For two items to be completely similar and equal in every way that can be known about in intuition is for them to be **congruous**. All geometrical construction of complete identity rests on congruity. This congruity of two combined motions with a third (as what is composed by the two) can never take place when the two are represented in a single space, e.g. in a single relative space. Hence each attempt to •disprove the Proposition on page 14 has failed because it is come up with merely mechanical solutions—saying how two motions m_1 and m_2 **combine with one another to produce** m_3 a third motion. Such attempts didn't prove that m_1 and m_2 **were identical with** m_3 and that because of this identity they could be presented in pure intuition *a priori.* [Kant wrote 'Each attempt to •prove the Proposition', but that must have been a slip.]

Remark 2

When a speed AC is termed 'double', this can only mean that it consists of two simple and equal speeds AB and BC (see the diagram on page 14). But if a 'double speed' is explained as 'a motion whereby a doubly great space is traversed in the same time', then something is being assumed that shouldn't be taken for granted, namely that two equal speeds can be combined in the same way as two equal spaces. It isn't obvious that a given speed consists of smaller speeds—that a speed is made up of slownesses!—in the way that a space consists of smaller spaces. The parts of the speed aren't external to one another, as the parts of the space are; and if a speed is to be considered as an amount, then the concept of its amount (·'How fast?'·) can't be constructed in the same way as the concept of the size of a space (·'How big?'·), because the former is intensive and the latter extensive. [Except for a passing mention (not included in this version), this is the first time Kant has used 'intensive' in this work. Examples: 'How severe was the pain?' and 'How hot is the water?' 'How fast did the train go?' ask about intensive magnitude, 'How long did the pain last?' and 'How much water is there?' and 'How far did the train go?' ask about extensive magnitude.] But the only way this 494
construction *can* be done is by putting together two equal motions, the motion of the body in one direction and the

equal motion of the relative space in the opposite direction. Two equal speeds can't be combined in one body except through external moving causes—e.g. a ship carries the body with one of these speeds while another moving force within the ship gives the body a second speed equal to the first. . . . So much for the *addition* of speeds to one another. But when it's a matter of subtracting one speed from another, it is easy enough to •think of such subtraction once we have the notion of a speed as an amount by *addition*; but it's not so easy to •construct the concept of this subtraction. To do this one must combine two opposite motions in one body—and how is that to happen? It can't happen if we work with only one space that doesn't move. 'Isn't the concept of opposite and equal motions of a single body in a single space simply the concept of *rest*?' No, it is not! What we get out of this is not the concept of rest but merely the fact that what we are trying to do is impossible. As I have already shown, the composition that is assumed in the proposition ·on page 14· to be possible has to be done by combining the motion of the body with the motion of the ·relative· space ·that contains it·. Finally, the composition of two motions whose directions enclose an angle: this also can't be thought of in the body by reference to a single space. We *can* make sense of there being a body which is acted on by a northward-pushing force and a westward-pushing one, which between them produce a movement of the body in the north-westerly direction. But that is the •mechanical account of the concept of this kind of composition, not the •mathematical construction of it. A mathematical construction has only to make intuitive what the combined movement *is*, not how it can be *produced* by
495 Nature or art through certain tools and forces. . . .

Remark 3

So there we have phoronomy—a pure doctrine not of •motion but of the •quantity of motion, in which matter is thought of wholly in terms of its mere movability. All it contains is this single proposition—·the one on page 14·—about the composition of motion, applied to the three kinds of cases I have discussed. And it only concerns straight-line motions, not motions along curves; because curved-line motion is continuously changing in direction, and there has to be a cause for this change, a cause that can't be merely space. People usually take the phrase 'composite motion' to refer only to the case where the directions of the motion enclose an angle; this. . . . hasn't done any harm to physics, because in physics all three kinds of combination can be adequately treated as versions of the third case, ·the enclosed-angle· one. If the angle enclosing the two given motions is thought of as infinitely small [i.e. as approaching 0 degrees], it contains the first case; and if the angle is represented as only infinitely little different from a single straight line [i.e. as approaching 180 degrees], it contains the second case. So all three of the cases I have listed can indeed be covered by the single familiar enclosing-an-angle formula. But a proper *a priori* grasp of the quantitative doctrine of motion isn't provided by that formula, and such a grasp is useful for many purposes. [Perhaps that last remark goes with Kant's saying that confining 'composite motion' to the enclosed-angle kind of case is harmful to 'the principle of the classification of a pure philosophical science in general'.]

[Kant ends this chapter with a needlessly difficult paragraph connecting the •three kinds of composition of motion with the •three categories—i.e. pure concepts of the understanding—that he lists under heading 'Quantity' in the *Critique of Pure Reason*. In that work the division is into •unity, •plurality, •totality (corresponding to propositions of the form •'Henry is a tyrant', •'Some husbands are tyrants', •'All weak husbands are tyrants'). Kant hopes to link that with phoronomy by speaking of the latter in terms of •unity of line and direction, the •plurality of directions in one and the same line, and finally the •totality of directions as well as of lines.]

[In that paragraph, the *Critique*'s Quantity trio are labelled first by *Größe* and then by *Quantität*. But in the *Critique* the only label is *Quantität*, whereas *Größe* is regularly used there for 'size' or 'magnitude'. Quite apart from questions of consistency, *Größe* just does mean 'size' or 'amount' or something like it, and has nothing to do with that one/some/all trio of categories; the only two places where Kant writes as though it were the right label for that trio is the paragraph reported above and in the list of category-trios on page 6. —Setting aside issues about the terminology of the *Critique* (which won't concern us much), the present version will mainly translate

> *Größe* by 'size' or 'magnitude' or 'amount' or by phrases using 'how much' or 'how strong' etc., and *Quantität* by 'quantity'.

The standard meanings of the German words are confirmed by Kant's uses of them: *Größe* stands for a universal—bigness, how-much-ness, something that a thing *has*; whereas *Quantität* stands for a particular portion—e.g. the portion of coffee that I drank a moment ago—this being something that a thing *is*. Both *Quantität* and 'quantity' can also be used to name a universal, but they have this other option, which is the one Kant sometimes employs. Quite often he uses *Quantität* to stand for a universal—i.e. as equivalent to *Größe*—and in those cases the relevant English word will have a subscript q, as in 'the amount$_q$ of matter in it' on page 47. Don't think or worry about this; it is put there just for the record. Just twice he uses *Größe* to mean 'quantity' in the non-universal sense.]

Chapter 2
Metaphysical Foundations of Dynamics

Definition 1

496 **Matter** is whatever is movable and fills a space. To **fill** a space means to resist every ·other· movable thing that tries to move into that space. A space that is not filled is an *empty* space.

Remark

This is, now, the dynamical definition of the concept of matter. This definition presupposes the phoronomic one [page 7] but adds to it a causal property, namely the capacity to resist a motion within a certain space. This property couldn't have any role in phoronomy, even when we were dealing with the motions of a single point in opposite directions. This *filling* of space keeps a certain space free from the intrusion of any other movable thing, whatever direction it is coming from. Now we must investigate what matter's all-around resistance is based on and what it *is*. Definition 1 makes it clear that we aren't talking about matter's resistance to •being pushed from one place to another (that's a mechanical phenomenon,
497 ·to be dealt with in chapter 3·), but only its resistance to •being squeezed into a smaller amount of space. The phrase 'occupying a space', i.e. being immediately present at every point in the space, is used merely to indicate the extension of a thing in space; and this concept of a thing's spatial extension or presence-in-space implies nothing about what if anything the thing does to resist other things that try to force their way into that space. It doesn't even rule out the possibility that something present in a given space acts causally to *attract* other movable things *into* that space. The concept might also apply to something that, rather than being an instance of matter *in* a space, is itself a space; because every space is an assemblage of smaller spaces, ·and one of them could be said to be *in* the larger space·. . . . Because it leaves all these possibilities open, the concept of *occupying* a space is broader and less determinate than the concept of *filling* a space.

Proposition 1

Matter **fills** a space not by its mere existence but by a special moving force.

Proof

Penetration into a space is motion. The cause of motion's becoming less, or even changing into immobility, is *resistance* to it. Now, the only thing that can be combined with a motion in such a way as to lessen or destroy it is another motion, in the opposite direction, of the same movable thing. [Kant adds '(phoronomic proposition)'; but what he has just said doesn't come from the Proposition on page 14. Perhaps it comes from the various proofs and comments relating to that Proposition.] Consequently, when a portion of matter x fills a space and thus resists all intrusion into that space by another portion of matter y, the resistance that it puts up against y's coming into the space is a cause of y's moving in the opposite direction. But our label for any cause of motion is 'moving force'. Consequently, matter fills its space not by merely being there but by ·exerting· moving force. [At the start of this paragraph, Kant says that the very first instant of a thing's movement is called *Bestrebung*, which can mean 'attempt' or 'endeavour' or the like. Like other early modern philosophers he used that term (or its equivalent in other languages) to stand for an *active tendency* that a body may have to move in a certain way. To say that thing has a *Bestrebung* to enter a given space is *not* to say •that it is consciously *trying* to move in, but it is to say more than merely •that it is in a state such that it will move in unless something stops it. From now on in this version, 'endeavour' will be used for *Bestrebung* (and not for anything else), but remember that it isn't a psychological term.]

Remark

Lambert and others used the rather ambiguous word 'solidity' to name the property of a portion of matter by which it fills a space; and they maintained that solidity must be possessed by every thing that exists (every substance), or at least by every thing in the external sensible world. According to their
498 way of thinking, a real thing x in a region of space must *by its very concept* carry with it this resistance: *the principle of contradiction* rules out there being anything else in the space containing x. But a portion of matter that is moving towards penetrating a space that already contains another portion of matter isn't pushed back by the principle of contradiction! The only way I can make sense of the suggestion that

> a contradiction is involved in a space's containing one thing x and being penetrated by another y

is by attributing to x a force through which *it* pushes back an external movable thing that approaches it. Here the mathematician (·Lambert·) has assumed, as an initial datum in constructing the concept of matter, something that doesn't admit of being further constructed. Well, he can indeed begin his construction with any datum he pleases, treating the datum as unanalysed; but he isn't entitled to block the route back to the first principles of natural science by analysing this datum as something wholly incapable of any mathematical construction.

Definition 2

Attractive force is the moving force through which a portion of matter can be the cause of another portion's moving towards it (or, equivalently, through which it resists another portion's moving away from it).

Repelling force is the moving force through which a portion of matter can be the cause of another portion's moving away from it (or, equivalently, through which it resists another portion's moving towards it).

[In English we have the verb 'move' both as transitive (as in 'She moved the jar to the end of the shelf') and intransitive as in 'You spoiled the picture: just as I clicked, you moved'. In the phrase translated as 'moving force' Kant is referring not to a force that moves-intransitive but rather to a force that moves-transitive; not a force that roams, but one that shoves. In fact, German doesn't have a verb that exactly matches the English intransitive 'move'. In the present version of this work, Kant is often translated as saying of some item that it 'moves'; but he does this with a German expression which would be mechnically translated as 'is moved'.]

Note

These are the only two moving forces that can be thought of, ·as I shall now prove·. In the context of questions about one portion of matter impressing some motion on another, the two portions must be regarded as *points*; so any transaction of that kind must be regarded as happening between two points on a single straight line. Now, there are only two ways for two points to move ·relative to one another· on a single straight line: either

- they approach one another, caused to do so by an attractive force; or
- they recede from one another, caused to do so by a repelling force. 499

Consequently, these two kinds of forces are the only ones we can make sense of; and all the forces of motion in material Nature must come down to them.

Proposition 2

(a) Matter fills its space by the repelling forces of all its parts, i.e. by its own *force of extension*, and **(b)** this ·repelling force· has a definite degree that can be thought of as smaller or greater to infinity. [This use of 'degree' translates what is almost the first occurrence of *Grad* in the original. From here on, *Grad*/degree will occur often; in Kant's usage it is firmly linked to the notion of *intensive* magnitude [see note on page 16]. We'll later see him writing about the degree to which a given portion of space is filled; this doesn't mean (extensive) *how much of* the space is filled but (intensive) *how strongly* the space is filled.]

Proof

(a) Matter fills a space only through moving force (Proposition 1), specifically by a moving force that resists the penetration, i.e. the approach, of other matter; and this is a repelling force (Definition 2). So matter fills its space only through repelling forces, and indeed through the repelling forces of all its parts. (Why '*all* its parts'? Well, try to suppose that some part x of a portion of matter doesn't exert repelling force. That means that the portion of space assigned to x is not *filled*, which means that that x isn't a portion of matter after all, but only a region of space contained within a portion of matter.) And the force of something that is extended by virtue of the repulsion of all its parts is a force of extension. [Kant adds in brackets that this is 'expansive' force—the first time this word has occurred in the work. We'll see a lot of it from now on.] Therefore, matter fills its space only by its own force of extension. **(b)** Given any particular force, it is conceivable that there should be a greater one. If for a given force F it was inconceivable that there should be a greater force, that would mean that F was the greatest conceivable force, which could make something travel an infinite distance in a finite length of time; which is impossible. ·Why 'an infinite distance'? Well, suppose that the best F can do is to make something travel N miles in a year, where N is a finite number; then it is conceivable that some force F+ should make a thing travel N+1 miles in a year, so that F+ would be greater than F. Where there's room for the thought 'greater distance' there's room for the thought 'greater force'·. Also, given any particular force, it is conceivable that there should be a lesser one. If that weren't so, there could be a force F such that a weaker force was inconceivable, which implies that the distance F could make a thing travel in a year was *zero*; meaning that it couldn't make anything move at all; meaning that F isn't a force of movement after all. (·The explanation of *zero* in this half of the proof of **(b)** can easily be worked out from the explanation of *infinity* in the first half·.) Putting **(a)** and **(b)** together: The force of extension through which every portion of matter fills its space has a degree that is never the greatest or smallest, but beyond which greater as well as smaller degrees can always be found. [Kant presumably means 'can be found in the realm of possibilities' = 'can be conceived', not 'can be found in the material world'. His later uses of 'can be found' will be translated without comment.]

Note 1

The expansive force of matter is also called *elasticity*. This 500
force is the basis for the filling of space as an essential property of all matter, so it is *basic*, not a consequence of any other property of matter. So all matter is basically elastic.

Note 2

Given any extensive force there can be found a greater moving force that can work against it and diminish the space that the extensive force is trying to expand. In this case the latter force is called a 'compressive' one. Thus, for any given portion of matter a compressive force can be found that can squeeze this matter into a smaller space than the one it is currently occupying.

Definition 3

A portion of matter x in its motion **penetrates** another portion y when by compression it completely abolishes the space of y's extension. [Kant's verb *aufhebt* apparently has to mean 'abolishes' in this context. But we'll see in a moment that what he means is that x *takes over* the space through which y was extended, depriving y of it.]

Remark
When an air-pump's piston is pushed ever closer to the bottom of the cylinder, the air-matter is compressed. If this compression could be carried so far that the piston came flat against the bottom with no air escaping, then the air-matter would be penetrated ·in the sense laid down in Definition 3·. For it is between two portions of matter that leave no space for it, so that it's to be met with between the bottom of the cylinder and the piston without occupying a space. This penetrability of matter by external compressive forces would be called 'mechanical', if there were such a thing—or indeed if such a thing were conceivable. I distinguish this impossible penetrability of matter from another kind of penetrability which is perhaps equally impossible. I may need to say a little about this second kind of penetrability later on. [We'll see that in this second kind of penetrability, which Kant will call 'chemical', x penetrates y by coming to share all y's space with y (see page 44). This is a much more natural meaning for 'penetrate' than the present 'mechanical' one.]

Proposition 3

501 **(a)** Matter can be •compressed to infinity, but **(b)** it can never be •penetrated by other matter, however great the latter's pressing force may be.

Proof A basic force through which a portion of matter tries to extend itself all through the space that it occupies must be greater when enclosed in a smaller space, and must be infinite when compressed into an infinitely small space. **(a)** Now, for any given extensive force that a portion of matter has, there can be found a greater compressive force that squeezes this matter into a smaller space, and so on to infinity. But **(b)** penetrating the matter would require its compression into an infinitely small space, and thus would require an infinitely strong compressive force; but such a force is impossible. Consequently, a portion of matter cannot be penetrated by the compression of any other portion of matter.

Remark
I have assumed at the start of this proof that the more an extensive force is constricted the more strongly it must resist. This might not hold for a •derivative elastic force, but it can be postulated of ·any •basic elastic force, i.e.· any elastic force that a portion of matter has essentially, just because it is matter filling a space. Expansive force exercised from all points toward all sides constitutes the very concept of elasticity. And the smaller the space in which a given amount of expanding force has to exercise itself, the more strongly the force must exercise itself at every point in the space.

Definition 4

The impenetrability of matter that comes from its resistance ·to being squeezed·—impenetrability that increases proportionally to the degree of compression—I call '**relative**'.
The impenetrability that comes from the assumption that 502
matter as such can't be compressed at all is called '**absolute**' impenetrability. The filling of space with absolute impenetrability can be called 'mathematical'; that with merely relative impenetrability can be called 'dynamical'.

Remark 1

According to the merely mathematical concept of impenetrability (which doesn't assume that any moving force is basically inherent in matter), no matter can be compressed except to the extent that it contains empty spaces within itself. So matter, just *as matter*, resists all penetration unconditionally and with absolute necessity. According to my discussion of it, however, impenetrability has a physical basis; for the extensive force makes matter itself, as something extended filling its space, first of all possible. But this force has a degree that can be overcome, so the space occupied by a portion of matter can be diminished, i.e. its space can be somewhat penetrated by a given compressive force; but complete penetration is impossible, because it would require an infinite compressive force. Because of all this, the filling of space must be regarded only as relative impenetrability.

Remark 2

In fact absolute impenetrability is nothing more or less than a *qualitas occulta*. [Kant here refers (in Latin) to the 'occult (= hidden) qualities' that were postulated by various mediaeval philosophers to 'explain' certain phenomena; by Kant's time, everyone agreed that these explanations were no good. There were two basic complaints about them: **(i)** They weren't derived from anything deeper or more general; they were always treated as basic, fundamental. **(ii)** Their 'explanations' were always slam-bang one-sentence affairs, with no complexity that might enable them to connect fruitfully with other explanations of other phenomena.] We ask '*Why* can't portions of matter penetrate one another in their motion?' and are given the answer 'Because they are *impenetrable*'! The appeal to *repelling force* is not open to this complaint. It is true that **(i)** this force also can't be shown to be possible through our giving a further analysis of it, so that we have to accept it as a fundamental force; but it doesn't **(ii)** lack helpful complexity, because it involves the concept of an •active cause and of •the laws of this cause in accordance with which the strength of the force can be measured by how strongly the space in question resists penetration.

Definition 5

Material substance is whatever it is in space that is movable on its own, i.e. separated from everything else existing outside it in space. The motion of a portion of matter whereby 503
it ceases to be a part ·of some larger portion of matter· is **separation.** The separation of the parts of a portion of matter is physical division.

Remark

The concept of substance signifies the ultimate subject of existence, i.e. everything that doesn't exist merely as a predicate [here = 'property'] of some other existing thing, ·in the way a blush exists merely as a property of a face, or a storm exists merely as a property of some wind and water·. Now, matter is the subject of everything *existent* in space; for besides matter no other spatial subject can be thought of except space itself; and the concept of space hasn't any content relating to existence, and merely contains the necessary conditions for things we can perceive through the external senses to have external relations to one another. So •matter—as what is movable in space—is •substance in space. Similarly every part of a portion of matter will also be a substance, because it too is itself a subject and not merely a predicate of other portions of matter; so every part of any portion of matter is itself a portion of matter. . . .

Proposition 4

Matter is divisible to infinity, and indeed into parts each of which is again matter.

Proof

Matter is impenetrable because of its basic force of extension (Proposition 3 [page 22]), but this force of extension is only the consequence of the repelling forces of each point in a space filled with matter. Now, the space that matter fills is mathematically divisible to infinity, i.e. its parts can be differentiated to infinity; although they can't be moved and so can't be pulled apart. . . . Now, in a space filled with matter every part of the space contains repelling force to hold at bay on all sides all the parts surrounding it, and hence to repel them and be repelled by them, i.e. to be moved to
504 a distance away from them. Hence every part of a space filled by matter is movable and is therefore separable by physical division from any of the other parts that are material substances. Consequently, every mathematical division of a region of space has corresponding to it a possible physical division—a pulling apart—of the substance that fills the region of space; and such mathematical divisions can be continued to infinity, so all matter is physically divisible to infinity—divisible indeed into parts each of which is itself also a material substance.

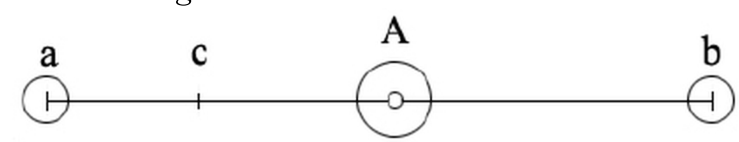
Remark 1

Proving the infinite divisibility of space is *far* from proving the infinite divisibility of matter unless one first shows that in every part of space there is material substance, i.e. separately movable parts. ·To see the need for this further premise, consider this position, which· a monadist might adopt:

> 'Matter consists of physical *points*, each of which—just because it *is* a point—has no separately movable parts, but nevertheless fills a region of space by mere repelling force. The region containing such a physical point is divided, but the substance acting in it—the physical point—is not divided.'

Thus, this monadist can have matter made up of physically indivisible parts while still allowing it to occupy space in a dynamical way, ·i.e. to occupy space by exerting force throughout it·.

But the proof I have given completely undermines this monadist dodge. My proof makes it clear that *every* point in a filled space must push back against whatever pushes in upon it. This •can be the case if the point contains a reacting subject that is separately movable and distinct from every other repelling point; and it's clear that it •can't be the case if all you have is a mere driving force exerting itself through a region of space. To get an intuitive grasp of this (and, therefore, of the proof I have given for Proposition 4), consider this diagram:



A is stipulated to be a monad whose sphere of repulsive force has the line aAb as a diameter. Then penetration of A's sphere of influence is resisted at the point a. But now consider a point c that is within the sphere, between a and A (there must be such a point, because space is infinitely divisible); and ask yourself what the state of affairs is at c. The answer is that *there must be at c something that holds A apart from a*:

> A force emitted from A can't make itself felt at a unless ·the contents of· those two points are kept apart; without that, they would penetrate one another ·so that the entire sphere would condense into a point·. 505

So something at c resists penetration by a and by A; so it repells the monad A at the same time as being repelled by it. And repelling is a kind of *motion*. So we get the result that c is something movable, ich means that it is matter. This shows that the filling of that sphere can't consist merely in a repelling force's being exerted throughout it by a one-point monad in its centre. On the contrary, the sphere must be filled with matter. (We are assuming, of course, that the argument about the point c could be repeated for *any* point within the sphere.)

Mathematicians represent the repelling forces of the parts of elastic portions of matter. . . .as increasing or decreasing in proportion to their distances from one another. The smallest parts of air, for instance, repel each other in inverse proportion to the distance between them, because their elasticity is inversely proportional to the spaces that they are squeezed into. Don't misunderstand the thought and mistake the language of the mathematicians by taking •something that necessarily belongs to *the process of constructing* the concept to be •something that applies to *the object* of the concept. ·Here's why they are different·. In the construction process, two things' being in contact can be represented as their being an infinitely small distance apart; and indeed the construction *has to* handle contact in that way in cases where a single quantity [*Quantität*] of matter, i.e. a single quantum of repelling forces, is represented as *completely* filling spaces of different sizes ·at different times·. For us to •have an intuitive sense of the expansion of a portion of matter to fill a larger space—·•that being what constructions are for·—we have to make use of the idea of an infinitely small distance. [See the note on 'idea' on page 9.] But if matter is infinitely divisible, there can't be any actual distance between any two ·nearest· parts; however much a portion of matter expands, it is still a continuum.

Remark 2

When mathematicians are just doing mathematics, they can ignore the tricks played by mistaken metaphysics. They can be sure of the obvious mathematical truth that space is infinitely divisible, without caring about objections that may be brought against this by foolish nit-pickers. But when they are ·not merely doing mathematics but· taking mathematical propositions that are valid for *space* and applying them to *substance filling space*, they have to submit what they are saying to purely conceptual tests, which means that they have to attend to metaphysics. Proposition 4 [page 24] is already a proof of this. For although matter is infinitely divisible *mathematically*, it doesn't follow that matter is *physically* divisible to infinity. Granted that every part of space is also a space, so that every part of space includes within itself parts that are external to one another, it doesn't follow that in every possible part of this filled space there is substance, which is separated from everything else and is independently movable. [Notice that Kant says 'filled space'—a phrase that he uses quite often to mean 'space filled with matter'. So the mathematicians' account of space as infinitely divisible stands firm even if the space in question is thought of as 'full of matter', provided (Kant warns) that this is left unexplained and (in particular) is not understood as meaning that every part of space contains *a material substance*. To the proposition that he is allowing the mathematicians to assert he might give the label 'the mathematical proposition of the infinite divisiblity of matter', setting this off against (a phrase that he *does* use) 'the physical proposition of the infinite divisibility of matter'.] So there has always been something missing from mathematical proof ·of the infinite divisibility of matter·, and there has been no guarantee that that proof could be securely applied in natural science. This gap has now been filled—by ·my proof of· Proposition 4 above. Now we have the *physical* proposition 506
of the infinite divisibility of matter; and when it comes to

metaphysical attacks on *that*, the mathematician must ·back off and· leave them entirely to the philosopher. When the philosopher tries to deal with these attacks, he ventures into a labyrinth that is hard enough to get through when he just approaches it philosophically; he can do without interference from mathematician! ·Here's a sketch of the labyrinthine problem· (stated for portions of matter, though it applies equally to regions of space):

> **(a)** A whole must already contain within itself all the parts into which it can be divided. Therefore **(b)** if matter is infinitely divisible, then it consists of infinitely many parts. But **(c)** a portion of matter can't possibly have infinitely many parts, because **(d)** the concept of *infiniteness* is the concept of *something that can't ever be wholly complete*, from which it follows that 'There are infinitely many of them, and they are all there, complete, settled' is self-contradictory.

That is the difficulty as it presents itself to the dogmatic metaphysician, who is thinking of wholes as *things in themselves*, the crucial point being that proposition **(a)** is true only of wholes considered as things in themselves. So we have to choose between two options:

> •Defy the geometer by denying **(1)** that space is divisible to infinity.
>
> •Annoy the metaphysician by denying **(2)** that •matter is a thing in itself and •space a property of a thing in itself, saying instead that matter is a mere appearance of our external senses and that space is just the essential form of matter, ·i.e. of that appearance·.

The philosopher is now squeezed between the horns of a dangerous dilemma. It's no use denying **(1)** that space is divisible to infinity; that's a mathematical result, and you can't get rid of it by tricky argument! But regarding matter as a thing in itself, and thus regarding space as a property of things in themselves, *is* denying **(1)**. So the philosopher sees himself as forced to depart from the assertion **(2)** that matter is a thing in itself and space a property of things in themselves—maintaining instead that space is only the *form* of our external sensible intuition [see note on page 8], so that matter and space are not things in themselves but only subjective modes of representation of objects that are *in themselves* unknown to us. Proposition **(2)** is common and commonsensical; the philosopher denies it only on the understanding that this will get him out of the difficulty about matter's being infinitely divisible yet not consisting of infinitely many parts. That matter consists of infinitely many parts can indeed be thought by reason, though this thought can't be constructed and made intuitable [see note on page 2]. If something x is •actual only by •being given in a representation, all you are given ·when you think of it· is what's met with in the representation, i.e. as far as the sequence of representations reaches. If something is an appearance that can be divided to infinity, what can we say about *how many* parts it has? Only that it has as many parts as we give it, i.e. as many as result from whatever division of it we choose to make. That's because the parts of something 507
that is merely an appearance exist only in thought, i.e. only in ·the thought of· the division itself. The division does indeed go on to infinity, but it is never *given* as infinite; so we can't infer that the divisible item contains within itself infinitely many parts ·that are things· in themselves existing independently of our representation of them. Why can't we? Because the division that can be infinitely continued is the division not •of the thing but only •of its representation. . . . A great man who perhaps contributes more than anyone else to the reputation of mathematics in Germany has several times rejected the impudent metaphysical claim to overturn what geometry teaches concerning the infinite divisibility of space.

[Who? Leibniz is a good guess (see below), except that the tenses in the foregoing sentence don't seem right for someone who had been dead for 70 years when Kant wrote this work.] His basis for this rejection was the reminder that space belongs only to the appearance of external things; but his readers didn't understand him. They took him to mean:

> **M:** Space is a thing in itself or a relation amongst things in themselves; but it appears to us, and the mathematicians ·aren't vulnerable to metaphysical attack because they· are talking only about space as it appears, ·not about actual space itself·.

What they should have understood him to mean is this:

> Space isn't a property of anything outside of our senses; it is only the subjective form of our sensibility. Objects of our external senses appear to us under this form, and we call this appearance *matter*. As for what these objects are like ·in themselves·—we know nothing about that.

According to the misinterpretation **M**, space was always thought of as a quality that things have independently of our power of representation, and the mathematicians ·were being criticised because they· thought of this quality only through common concepts (i.e. thought of it confusedly, for appearance is commonly thought of confusedly). This meant that according to **M** the geometricians had used a •confused representation of space as their basis for a mathematical proposition—asserting the infinite divisibility of matter—which presupposes the highest •clarity in the concept of space. Thus the door was left open for the **M**-accepting metaphysicians to bring clarity into this concept of space (they thought!) by supposing that space is made up of points and matter is made up of simple parts, ·i.e. parts that did not in their turn have parts·. This error was based on another misinterpretation—namely a misunderstanding of the monadology of Leibniz, which they saw as trying to explain natural appearances whereas really it is a platonic concept of the world. There's nothing wrong with Leibniz's concept ·of the world as a system of sizeless monads·, as long as the world is being regarded not as •an object of the *senses* but as •a thing in itself, i.e. as merely an object of the *understanding*, though it is the foundation of the appearances of the senses. [From here down to the next mention of Leibniz, this version expands on Kant's words in ways that the ·small dots· convention can't easily signify.] Now, any composite thing made up of things in themselves must certainly consist of *simple* things, because a composite *thing in itself* can't exist except as an upshot of the existence of its parts, *all* its parts, right down to the smallest ones that don't *have* parts. But a composite thing that is *an appearance* doesn't consist of 508
simple things, because its parts exist only as upshots of a division of the thing; so that they, rather than existing independently of the composite thing of which they are parts, exist only *in* that composite thing. For a thing in itself x:

> x exists as an upshot of the putting together of its parts;

whereas for an appearance y:

> y's parts exist as upshots of the division of y.

So it seems to me that Leibniz didn't intend to explain space in terms of an order of simple entities side by side, but rather to claim that this order *corresponds to* space while still belonging to a merely intelligible world that is unknown by us. And this is to assert just what I said elsewhere [in the *Critique of Pure Reason*], namely that space along with matter . . . . doesn't make up the world of things in themselves but only the appearance of such a world, and that what space itself *is* is only the form of our external sensible intuition.

Proposition 5

The possibility of matter requires a force of attraction, as the second essential basic force of matter.

Proof

Impenetrability, as the fundamental property of matter through which it first reveals itself as something real in the space of our external senses, is nothing but matter's power of extension (Proposition 2). Now, an essential moving force by which parts of matter pull away from one another cannot

> **(1)** be limited by itself, because such a force works on matter to drive it towards continuously expanding the space that it occupies;

and it cannot

> **(2)** be kept within limits by space itself. Why not? Because the most that space can do is to bring it about that when the volume of a portion of matter is increasing the extensive force becomes correspondingly weaker; such weakenings can go on to infinity—·the strength of a force is continuous·—but they can't reach zero, which is to say that space can't bring it about that the extensive force stops.

Therefore, if matter were driven only by its repelling force (the source of its impenetrability), with no other moving force counteracting this repelling one, there would be nothing to limit matter's extension; every portion of matter would disperse itself to infinity, so that no assignable quantity [*Quantität*] of matter would be found in any assignable ·region of· space. Consequently, if there were only repelling forces in matter, all regions of space would be empty—so that strictly speaking there wouldn't *be* any matter! [The thought is this: Let R be a region of space measuring a billion cubic kilometers, and let M be a portion of matter weighing a billionth of a gram: if matter expanded *infinitely*, there wouldn't be as big a portion of matter as M in a space as small as R, because that amount of matter would have been spread still more thinly through a still larger region of space. Repeat the argument, making M ever smaller and R ever larger; you will *always* have too much matter for that amount of space.] For matter to exist, therefore, it must have compressive forces opposed to the extensive 509
forces. 'Might not the force that keeps material portion x within limits be the ·expansive· force of a different portion y?' No, that can't be the basic account of the situation, because this 'different portion y' can't exist as matter unless some compressive force is acting upon *it*. So we have to assume that matter has a basic force acting in an opposite direction to the repelling force; this force must tend to bring things closer to one another, which is to say that it must be an *attractive* force. Now, this attractive force is needed for any matter to be possible, so it is more basic than any differences between kinds of matter; and therefore it must be attributed not merely to some one species of matter but to all matter. Thus, a basic attraction belongs to all matter as a basic force that is part of its essence.

Remark

We need to look more closely into what happens in our thinking when when we move from •one property ·that is *contained in*· the concept of matter to •a radically different property that equally *belongs to* the concept of matter without being contained in it. If attractive force is basically required for matter to be possible, why don't we use it, along with impenetrability, as the primary sign of matter? Impenetrability is given immediately with the concept of matter, while attraction isn't thought *in* the concept but only associated with it by inference—what's going on here? You might think: 'Well, our senses don't let us perceive attraction as immediately as repulsion and the resistance of impenetrability'—but that doesn't properly answer the

question. Suppose that we *could* perceive attraction as easily as repulsion: our understanding would still choose to differentiate space from matter—i.e. to designate substance *in* space—in terms of the *filling* of space (otherwise known as *solidity*). Attraction, however well we perceived it, ·couldn't do the job. It· would never reveal to us any portion of matter with a definite volume and shape. All it could reveal to us would be our perceiving organ's being tugged towards a point outside us, namely the central point of the attracting body. [Translated more strictly, Kant speaks not of the organ's being tugged but of its 'endeavouring' to reach that external point. Either way, it is initially surprising, but it is not unreasonable. How do we perceive repelling forces? By feeling ourselves being pushed away from things. So how would we (if we could) perceive attractive forces? By feeling ourselves being pulled towards things! This interpretation presupposes that the 'perceiving organ' is the perceiver's body, the 'organ' of the sense of touch.] That experience wouldn't reveal to us any material things with definite sizes and shapes, because the only way the attractive force of all parts of the earth could affect us is exactly the same as if that force were concentrated entirely in the centre of the earth and this point alone were tugging us; similarly with the attraction of a mountain, or of a stone, etc.—the pull would always be to the central point, and would give no sense of the relevant body's shape or
510 size of even its location. (·Why not its location? Because· although we would be able to perceive the *direction* of the attraction, as it is perceived in our experience of weight, we wouldn't know how far away it was in that direction.) The attracting point would be unknown, and I don't see how it could even be discovered through inferences unless we already had perceptions of matter as filling space, ·i.e. as having repelling force·. This makes it clear that our first application of our concepts of *size* to matter. . . .is based only on matter's space-filling property. Through our sense of touch this property tells us the size and shape of an extended thing, thus creating the concept of a determinate object in space—a concept that underlies everything else that can be said about this thing. No doubt this is what explains the fact that although there are very clear proofs that attraction must belong to the basic forces of matter just as much as repulsion does, there are people who strenuously reject attractive forces and won't allow matter to have any forces except those of impact and pressure (both by means of impenetrability). 'What space is filled by is substance', they say; and this is correct enough, ·but its correctness has led these people astray·. The substance that they talk about reveals its existence to us through the sense by which we perceive its impenetrability, namely the sense of touch; so it reveals its existence only through the contact of one portion of matter with another—a process that starts with *collision* and continues with *pressure*. And because of this it seems as though the only way for one material thing to act immediately on another is by colliding with it or putting pressure on it—these being the two influences that we can immediately perceive. Whereas it's very hard for us to think of *attraction* as a basic force, because it doesn't give us any sensation at all, or anyway no definite object of sensation.

Proposition 6

Matter isn't made possible by mere attraction, without repulsion.

Proof

Attractive force is the moving force of matter whereby one material thing gets another to approach it. If every part of the material world exercises such a force, all those parts are led to cluster together, thus shrinking the region of space that they jointly occupy. Now, the only thing that can block the action of a moving force is a moving force opposed to it; and the force that is opposite to attraction is the force of repulsion. If *that* didn't exist, there would be nothing to stop the force of attraction from pulling portions of matter together closer and closer, constantly shrinking the region of space containing matter. There would be no such thing as

> two material things so close together that repelling forces block them from coming even closer,

so that the force of attraction would eventually pull material things closer and closer together until they shrank into a mathematical point; and at that stage space would be empty, i.e. wouldn't contain any matter. So matter is impossible through mere attractive forces without repelling ones. [Notice the elegant shape of Kant's arguments about the two kinds of force. Allow only repulsion/expansion and matter is spread so widely and thus thinly that it disappears; allow only attraction/contraction and matter is packed so densely that it is all contained in a single point and disappears from all space except that point.]

Note

Any property that is required for something to be intrinsically possible (·whether or not possible in relation to other things·) is itself an essential element of that intrinsic possibility. So repelling force belongs to the essence of matter as much as attractive force does—the two can't be separated in the concept of matter.

Remark

I had first to consider the forces of repulsion and attraction •separately, in order to see what each on its own could contribute to the presentation of matter. The upshot was an *a priori* proof that they are both present, •united, in the general concept of *matter*. We found that space remains empty, with no matter to be found in it, unless *both* these forces are at work in it. ·Why only these two forces—why
511 only repulsion and attraction?· Because they are the only ones that are thinkable.

Definition 6

Contact in the physical sense is the immediate action and reaction of impenetrability. The action of one portion of matter on another when there is no contact between them is **action at a distance**. When this action at a distance occurs without the mediation of matter lying between the
two portions of matter it is called **unmediated** action at a 512
distance, or the action of portions of matter on one another through empty space. [Kant's word *unmittelbar* is usually translated as 'immediate'; and that is not incorrect. But it's natural for us to think of x's 'immediate' influence on y as ruling out not only **(a)** any mediating *thing* between them but also **(b)** any distance between x and y as well. Therefore, in cases where Kant is ruling out **(a)** and emphatically *not* ruling out **(b)**, 'unmediated' will be used instead.]

Remark

Contact in the mathematical sense of the word is the shared boundary of two regions of space—so it isn't *in* either of them. So straight lines can't be in contact (in this sense) with one another: when two straight lines have a point in common, that is because they *intersect*, and their common point belongs to each of them. But a circle and a straight line can be in contact at a point, and so can a circle and another circle; two planes can be in contact at a line, and two solids can be in contact at a plane. Mathematical contact lies at the basis of physical contact, but it doesn't *constitute* it. To get from the concept of mathematical contact to that of physical contact you have to add the thought of a dynamical relation—not that of the attractive forces but the relation of the repelling ones, i.e. of impenetrability. Physical contact is the two-way interaction of repelling forces at the common boundary of two portions of matter.

Proposition 7

The attraction that is essential to all matter is an unmediated action through empty space of one portion of matter on another.

Proof

The possibility of matter as *the thing that fills a space in a determinate degree* depends on the basic attractive force, and so the possibility of physical contact between portions of matter also depends on it. [Until now Kant hasn't spoken explicitly of regions as being filled to a greater or lesser degree, more or less *intensively* filled; but he has done so implicitly, by saying that the repelling force that constitutes space-filling is a matter of degree, i.e. can be more or less strong at a given point. This concept of *the degree to which a given region of space is filled* will be crucially important in what follows.] Thus, physical contact presupposes the attractive force, so the force can't depend on there being physical contact. Now, the action of a moving force that •doesn't depend on any contact •doesn't depend either on the filling of space between the moving thing and the thing moved, ·because 'the space between x and y is filled' is equivalent to 'from x to y there is a series of portions of matter, each in contact with the next'·. This means that such action must occur without the intervening space being filled, and so it's action that operates through empty space. Therefore the basic essential attraction of all matter is an unmediated action of portions of matter upon one another through empty space.

Remark 1

513 It is completely impossible to make any basic force *conceivable*, i.e. to present one or more other forces that somehow give rise to it. Just because it is a *basic* force it can't be derived from anything.

[This use of 'conceivable' may seem odd. It comes from the fact that Kant is running the proposition

> The concept of attraction can't be analysed into simpler or more basic concepts

in the same harness as the proposition

> The attractive force can't be shown to be derived from and dependent on some more basic forces.

On page 40 we shall find Kant inferring from propositions of the type

> The. . . force can't be shown to be derived from and dependent on some more basic forces

the corresponding propositions of the form

> It isn't possible for us to comprehend the possibility of the. . . force.

He regards this as an inevitable drawback of any theory that postulates basic forces; but we'll see that it's a drawback he is willing to put up with because of the advantages of that kind of theory.] But the basic attractive force isn't even slightly *more* inconceivable than the basic force of repulsion. The difference is merely that the basic attractive force doesn't offer itself so *immediately* to our senses as impenetrability—the repelling force—does in giving us concepts of determinate objects in space. Because it's not •felt but only •inferred, the attractive force gives the impression of being ·not a •basic force but· a •derived one, as though repulsion were the upshot of a hidden play of ·more basic· moving forces. But when we take a closer look at attraction, we see that it can't be derived from any source, least of all from the moving force of portions of matter through their impenetrability, because its action is exactly the opposite of impenetrability. The most common objection to unmediated action at a distance is the claim that *a portion of matter can't directly act at a place if it isn't there.* ·But· when the earth directly influences the moon to come closer, it is acting *unmediatedly* on a thing thousands of miles away; and the space between the earth and the moon might as well be regarded as entirely empty, because even if there is matter there it has no effect on the attraction. So the earth acts directly in a place without itself being there. That may *seem* to be self-contradictory, but it isn't. The truth of the matter in fact is that whenever *anything* in space acts on anything else, it acts in a place without itself being in it! If something were to act in the same place where it itself is present, then it wouldn't be acting on anything outside it, but only on itself. For a thing x to be 'outside' a thing y is for x to be in a place that doesn't have y in it. If the earth and the moon touched each other, the point of contact would be a place that has neither the earth nor the moon in it.... It wouldn't even have anyI *part* of either the earth or the moon in it, because this point lies at the boundary of the two filled regions, and this boundary isn't a part of either of them. It follows from this that the ·widely accepted· proposition that

•portions of matter cannot unmediatedly act on each other at a distance

amounts to the proposition that

•portions of matter can't unmediatedly [*unmittelbar*] act on each other without the intervention [*Vermittelung*] of the forces of impenetrability.

This amounts to saying that repelling forces are the only ones by which portions of matter can be active, or at least that they must be involved when portions of matter act on one another; which implies that the force of attraction is either •impossible or •always dependent on the action of repelling forces; and there is no basis for either of those assertions. The ·widespread· misunderstanding of this matter is a result
514 of confusing •the mathematical contact of regions of space with •their physical contact through repelling forces. [The rest of this paragraph expands Kant's words in ways that the ·small dots· convention can't easily signal.] For x to attract y unmediatedly and without contact is for this to be the case:

(1) x and y come closer together in accordance with a constant law of the form 'If two portions of matter have relation R_1 between them, they move towards one another'.

And for x to repel y unmediatedly and without contact is for this to be the case:

(2) x and y move away from one another in accordance with a constant law of the form 'If two portions of matter have relation R_2 between them, they move away from one another'.

Now, there is not the slightest difficulty about supposing that repelling force doesn't come into R_1 and that attractive force doesn't come into R_2. These two moving forces are wholly different in kind, and there's not the slightest basis for claiming, of either of them, that it depends on the other and isn't possible without the intervention of the other.

Remark 2

Attraction between two things that are in contact can't result in any motion. Why not? Because for two bodies to be in contact is for the impenetrability of each to act against the impenetrability of the other, and that impedes all motion. So there must be *some* unmediated attraction without contact, i.e. unmediated attraction at a distance. To see why, suppose that it is not so, and see where you get. We have two bodies that are approaching one another, without unmediated attraction being at work. In that case, the situation must be that they are being *pushed* towards one another by forces of pressure and impact. This is only apparent attraction, as against true attraction in which repelling forces have no role at all. But even such an apparent attraction must, deep down, involve true attraction, because the portions of matter whose pressure or impact is at work wouldn't even *be matter* if they didn't have attractive forces (Proposition 5 [page 28]). So the attempt to ·get rid of true attraction and· explain all phenomena of approach in terms of apparent attraction moves in a circle.

There is a view about Newton that is widely accepted, namely:

> He didn't see any need for his system to postulate unmediated attraction of portions of matter. Behaving strictly like a pure mathematician, he •kept right out of this issue, •left the physicists completely free to explain the possibility of such attraction in whatever way they thought best, and •avoided mixing up his propositions with their play of hypotheses.

But how could he establish the proposition that *the universal attraction of bodies—across a given distance—is proportional to the quantity* [*Quantität*] *of matter in the bodies* if he didn't assume that it's an essential feature of matter as such, ·matter simply *qua* matter·, that it exercises this motive force? For when one body pulls another, their approach to one another (according to •the law of the equality of
515 reciprocal action) must always occur in inverse proportion to ·the quantity of· the matter in those bodies—and it makes no difference what *kinds* of matter are involved. Still, •this law is not

> a principle of •dynamics, i.e. a law about the distribution of attractive forces,

but rather

> a law only of •mechanics, i,e, a law about the motions that attractive forces cause.

And not just *attractive* forces; it is valid for moving forces generally, of whatever kind. ·Here is an illustrative example·:

> A magnet x is attracted by an exactly similar magnet y on two occasions: on one occasion there are just the two magnets, on the other occasion magnet y is enclosed in a wooden box that weighs twice as much as y does. On the second occasion, y-plus-box will impart more relative motion to x than y alone did on the first occasion, despite the fact that the wood, which contributes to the quantity [*Quantität*] of the matter in y-plus-box, adds nothing at all to y's attractive force and exerts no magnetic attraction.

Newton ·regarded attraction as something that all matter, of whatever kind, must have. He· wrote:

> 'If the ether or any other body had no weight, it would differ from any other portion of matter only in its *form*, so that it could be transformed little by little through a gradual change of this form into a portion of matter of the heaviest kind on earth; and conversely the heaviest kind could become weightless through a change of its form. This is contrary to experience' and so on. [Newton's *Principia* II.vi.cor.2]

Thus he didn't exclude even the ether (much less other kinds of matter) from the law of attraction. If Newton held that the approach of bodies to one another was a case of mere apparent attraction, created ·somehow· by impact, what kind of matter would he be left with to provide the impact? So you can't claim this great founder of the theory of attraction as your predecessor, if you take the liberty of replacing the •true attraction that he did maintain by an •apparent attraction that forces you to explain the appproach of bodies in terms of impact. 'What causes the universal attraction of matter?' Newton declined to get into any hypotheses to answer this question; and he was right to do so, because the question belongs to physics or metaphysics, not mathematics. It's true that in the preface of the second edition of his *Optics* he says: 'And to show that I do not take gravity to be an essential property of bodies, I have added one question concerning its cause' and so on [Kant quotes this in Newton's Latin]. Well, perhaps he shared his contemporaries' shock at the concept of *basic attraction*, and was led by this to be at variance with himself. ·There can be no question of taking that remark from the *Optics* as his most fundamental and most considered view, because· he held that the attractive forces that two planets. . . .exercise on their satellites (mass unknown), when at the same distance from those satellites, are proportional to the quantity [*Quantität*] of the matter in the two planets; and he absolutely could not say this unless he assumed that *just by being matter* they had attractive force, in which case all matter must have it.

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Definition 7

A **superficial force** is a moving force by which portions of matter can directly act on one another only at the common surface of their contact; a **penetrating force** is a moving force by which one portion of matter can directly act on the parts of another that are not at the surface of contact.

Note

The repelling force through which matter fills a space is a mere superficial force. That is because the parts touching each other limit one another's sphere of action; the repelling force can't move any more distant part except by means of those lying between. . . . On the other hand, no intervening matter limits an attractive force. That kind of force enables a portion of matter to •occupy a region of space without filling it [see Remark on page 20]; and to •act through empty space upon other distant portions of matter, without this action's being limited by any intervening matter. That is how we must think of the basic ·force of· attraction that makes matter itself possible. So it's a penetrative force, and for that reason alone it is always proportional to the quantity [*Quantität*] of the matter.

Proposition 8

The basic attractive force, on which the very possibility of matter depends, reaches out directly from every part of the universe to every other part, to infinity.

Proof

Because the basic attractive force. . . .is essential to matter, every portion of matter has it. Now, suppose there were a distance beyond which the force of attraction didn't reach: 517
what could explain this limitation of the sphere of its efficacy? It would have to be explained either **(a)** by the matter lying within this sphere or **(b)** by the sheer size of the sphere. It couldn't be **(a)**, because this attraction is a penetrative force, which acts unmediatedly at a distance; it goes across every region of space as though the space were empty, unaffected by any intervening portions of matter. And **(b)** can't be right either. Every case of attraction involves a moving force that has a degree ·of strength·, given any such degree a smaller one is thinkable, and then one smaller than that. . . and so on to infinity. Now, the great distance between two portions of matter would reduce the strength of the attraction between them—reducing it in inverse proportion to the amount of the diffusion of the force—but it wouldn't destroy the attractive force between them completely. So there is nothing that could bring about a limit to the sphere of efficacy of the basic attraction of any part of matter, so this attraction reaches throughout the universe to infinity.

Note 1

We have here a basic attractive force—a *penetrating* force—which is exercised

•by every portion of matter (in proportion to its quantity [*Quantität*] of matter), •upon all portions of matter, •across any possible distance.

From this force, in combination with the opposing repelling force, it must be possible to derive the limitation of the repelling force and hence the possibility of the filling of a region of space to a determinate degree. And in this way the dynamical concept of matter as *what is movable, and fills a region of space to some determinate degree* can be constructed. This construction requires a law governing how basic attraction and basic repulsion relate to one another at various distances. Finding this relation is a purely mathematical problem, because the relation rests solely on •the opposite directions of these two forces (one drawing points together, the other pushing them apart) and on •the size of the space into which each force diffuses itself at various distances; metaphysics has nothing to do with this. If the attempt to construct matter in this way meets with failure, that won't be the fault of metaphysics. Its only responsibility
518 is for the correctness of the elements of the construction that reason leads us to; it isn't responsible for the insufficiency and limitedness of our reason in doing the construction.

Note 2

Each portion of matter succeeds in being a determinate material thing only by filling a region of space with a determinate degree of repelling force; and such a filling of a determinate region of space can happen only through a conflict between a basic attraction and the basic repulsion. Now, the attraction involved in this filling of a determinate region of space may arise either ·internally· from •the attractions that the parts of the compressed matter exert on one another or ·externally· from •the attraction exerted upon this compressed matter by all the matter of the world. The basic attraction is proportional to the quantity [*Quantität*] of matter, and it reaches to infinity. So the only way a determinate region of space can be filled by matter is through matter's infinitely-reaching attraction; such a determinate degree of the filling of space can then be imparted to every portion of matter in accordance with the degree of its repelling force. The action of universal attraction—exercised by all matter directly on all matter and at all distances—is called *gravitation*; the endeavour [see long note on page 19] to move in the dominant gravitational direction is *weight*. The action of the universal repelling force of the parts of each portion of matter is called its basic *elasticity*. Weight involves an external relation, while elasticity is internal. These two are the only *a priori* comprehensible universal characteristics of matter; ·they are *a priori* graspable because· they are the foundations on which rests the very possibility of matter. When *cohesion* is explained as the reciprocal attraction of portions of matter that are in contact with one another, it doesn't belong to the possibility of matter in general and therefore can't be known *a priori* to be bound up with matter. This property ·of cohesion through contact· would be physical, not metaphysical, so it wouldn't belong to our present considerations.

Remark 1

I can't forbear adding a small preliminary remark for the sake of any attempt that may be made toward such a possible construction.

(1) Let F be some force—*any* force—that acts unmediately at different distances, with the amount of moving
force that it exerts at any given point being limited only 519
by how far it had to travel to reach that point. However much or little space F is spread through, the total amount of it is the same; but the intensity of its action upon a given point x will always be inversely proportional to the space F had to get through to reach x. Think of light being propagated from a point P, surrounded by a series of spheres each with P as its centre. The total amount of light falling on any sphere is the same as the total amount falling on

any other; but the amount of light falling on (say) a square inch of one sphere will be greater than the amount falling on a square inch of a larger sphere. And that's how it is with all other forces, and the laws according to which these forces must diffuse themselves, either in two dimensions or in three, in order to act according to their nature upon distant objects. If you want to do a drawing of the diffusion of a moving force from one point, it is better *not* to do it in the ordinary way (as in optics, for example), namely by means of straight-line rays diverging from a central point. However many lines you put into such a diagram, they'll get further apart the further they get from the central point; so they can never *fill* the space through which they pass or (therefore) fill the surface that they reach. This makes them a source of troubles that can be avoided if we ·get rid of straight-line rays, and· think of the situation merely in terms of the size of the whole spherical surface that is to be uniformly illuminated by the same quantity [*Quantität*] of light, so that—quite naturally—the intensity of illumination of any given area of a surface is inversely proportional to the size of the whole surface; and similarly with every other diffusion of a force through spaces of different sizes.

(2) If the force is an unmediated *attraction* at a distance, the lines of the direction of the attraction must be represented as rays not •diverging from the attracting point but, rather •converging at the attracting point from all points of the surrounding spherical surface. Why? Because the line of direction of the motion to this point—a point that *causes* the motion and *is its goal*—assigns the points *from* which the lines must begin, namely from all points of the surface. These lines get their direction from this surface to the attracting centre ·of the sphere·, and not vice versa. For only the size of the surface determines how many lines there are; the centre leaves this undetermined.[2]

(3) If the force is an unmediated repulsion by which a 520
point. . . .fills a space dynamically, and if the question is

> What law of infinitely small distances (here = *contacts*) governs how a basic repelling force acts at different distances?. . . .

then it is even further from being correct to represent this force by diverging rays of repulsion coming from the repelling point, even though the direction of the motion has this point as its starting-point. That's because the space in which the force must be diffused in order to act at a distance is a corporeal space that is to be thought of as filled. There's no way of mathematically representing how a point can dynamically fill a space; and the repelling force of a corporeally filled space can't possibly be represented by diverging rays coming from a point. What we must do, rather, is to assign a value to the repulsion at various infinitely small distances of
these mutually repelling points simply in inverse proportion 521
to the ·volumes of the· corporeal spaces that each of these points dynamically fills, so that the value will be in inverse

[2] It's impossible to represent surfaces at given distances as wholly filled with the action of lines spreading out from a point like rays, whether the action is illumination or attraction. Draw the situation in that way and you make it look as though the inferior illumination of a distant spherical surface consists in its having relatively large *un*illuminated and widely spaced illuminated ones! Euler's hypothesis ·that light consists of waves, not streams of particles· avoids this inconvenience, but at the cost of making it harder to get a conception of the rectilinear motion of light. [The footnote goes on at some length, recommending that we think of light as consisting not of waves or of straight-line streams of particles but rather an infinitely divisible *fluid*. Kant seems to acknowledge that there is no convenient way to *draw* this account of the matter; and recommends that we resort to the device of straight-line rays but only after getting firmly and clearly in mind what the truth is, so as not to be misled by the lines.]

proportion to the cube of the distances of these points from one another. . . .

(4) So the basic attraction of matter would act in inverse proportion to the square of the distance—*any* distance—while the basic repulsion would act in inverse proportion to the cube of the infinitely small distances. It's that action and reaction of the two fundamental forces that make ·a portion of· *matter* possible, by filling its space to a determinate degree. The point is that as parts move closer together the •repulsion between them increases faster than the •attraction does; and that sets a limit to the approach—the limit at which the available attractive force loses out to the available repelling force—and that limit determines *how* intensely the space is filled.

Remark 2

I'm well aware of the difficulty about this way of explaining the possibility of *a portion of matter* ·considered as separate from other portions of matter·. It consists in the fact that if a point can't unmediatedly [see note on page 30] drive another point by repelling force without at the same time filling the whole intervening corporeal space with its force, then it seems to follow that this ·intervening· space must contain several driving points. That conflicts with the hypothesis ·of the discussion, namely that we are talking here about action *at a distance*·, and it was ruled out above through the label 'sphere of repulsion of the simple in space'. [Ruled out *where*? Kant cites Proposition 4, but that seems wrong. Definition 6 is better, though neither there nor anywhere else has he spoken of 'the repulsion of *the simple*'.] But we should distinguish •the *concept* of an actual region of space, which could exist, from •the mere *idea* of

> a region of space that •is entertained in thought only for the purpose of determining how various given regions are inter-related, but •isn't in fact a region of space.

[See note on *Idee* on page 9.] In the case cited of a supposed physical monadology, there were to be actual spaces that were filled by a point dynamically, i.e. through repulsion; for they existed as points before any possible production of matter from these points, and through the proper sphere of their activity they *fixed* the part of the space to be filled that could belong to them. In this physical monadology, therefore, matter can't be regarded as infinitely divisible and as a continuous quantum, because the parts that unmediatedly repel one another are at a determinate distance from one another (the sum of the radii of their spheres of repulsion); whereas the thought of matter as a continuous quantity [*Größe*] doesn't allow for *any* distance between the unmediately repelling parts, or, therefore, for any increase or decrease of the spheres of their unmediated activity. However, portions of matter can expand or be compressed (like the air), and ·within the framework of the physical monadology· this can be represented in terms of increase and decrease of the distance between their nearest parts. But ·in actual fact· the closest parts of a continuous portion of matter touch one another, even when it is being expanded or compressed; so their distances from one another have to be thought of 522
as infinitely small, and this infinitely small space must be understood to be filled in a greater or lesser *degree* [see note on page 21] by their force of repulsion. But two things' •having an infinitely small space between them is their •being in contact. Hence it is only the idea of space that enables us to intuit [= 'see in our mind's eye'] the expansion of matter as a continuous quantity [*Größe*], although it can't actually be conceived in this way. Thus, when it is said that the repelling forces that two parts of matter unmediatedly exercise on one another are

•in inverse proportion to the cube of the distance between them,

this means only that they are

•in inverse proportion to the corporeal spaces that one thinks of between the parts,

though in fact the parts are immediately in contact (which is why we have to call the distance between them 'infinitely small' so as to distinguish it from every actual distance). We mustn't raise any objection to a concept itself because of difficulties in the construction of it or rather in the misinterpretation of the construction of it. . . .

The universal law of dynamics would in both cases be this:

•The action of the moving force that one point exerts on each other point external to it is inversely proportional to •the space through which that moving force has had to spread in order to act unmediatedly upon the other point at the given distance.

From the law that the parts of matter basically repel one another in inverse cubic proportion to their infinitely small distances, there must necessarily follow a law of the expansion and compression of these parts that is entirely different from Mariotte's law regarding the air. Mariotte's law proves that the forces causing the closest parts of the air to move away from one another are in inverse proportion to the distances between parts (Newton proves this in the scholium to Proposition 23 of Book II of the *Principia*). But the expansive force of the parts of the air can't be an example of the action of *basic* repelling forces. Why not? Because this expansive force comes from *heat*, which compels the proper parts of the air (which, incidentally, are at *actual* distances from each other) to move away from one another, doing this, apparently, by *vibrations*. . . . But the laws of the communication of motion through the vibration of elastic portions of matter make it easy to conceive that these ·heated-air· vibrations give to the air's parts a force that •causes them to move away from one another and •stands in inverse proportion to the distances between the parts. [The phrase 'communication of motion' is a common translation of the German *Mitteilung der Bewegung*. It would be closer to the German to put 'sharing of motion', but we would have to remember to liken this to 'thank you for sharing that news with me' rather than to 'thank you for sharing your cake with me'. Or we might use 'the passing on of motion'; but on page 56 Kant writes about those who thought of the *Mitteilung der Bewegung* as a literal passing over of some motion, from one body that loses it to another that gains it. That is one *theory about* this phenomenon; so terminology that strongly suggests it can't be used as a *neutral name for* the phenomenon.] But let me explain: I do *not* want my exposition of the law of basic repulsion to be seen as essential to the aim of my metaphysical treatment of matter. All I needed for that 523
treatment was to present the filling of space as a dynamic property of matter; and I don't want that to be mixed up with the disputes and doubts that might arise from ·further details of· my exposition.

GENERAL NOTE ON DYNAMICS

Looking back over everything I have said about the metaphysical treatment of matter, we find that the treatment has dealt with

(1) what is real in space (otherwise known as what is 'solid') in its filling of space through repelling force;

(2) what relates in a negative way to the real in space . . . ., namely, attractive force, ·which negates the real in space in the sense that· if this attractive force were left to itself it would permeate the whole of space and completely abolish everything solid;

(3) the limitation of **(1)** by **(2)**, yielding an empirically accessible degree of the **filling of space.**

So we see that the quality of matter has been completely dealt with under the headings of *reality*, *negation*, and *limitation*. When I say 'completely dealt with', I mean that the treatment contains everything needed for a metaphysical dynamics. [The terms 'reality' etc. are Kant's labels for the categories of Quality in his *Critique of Pure Reason*.]

GENERAL REMARK ON DYNAMICS

In what I am about to say, I use 'real' [German *real*, from Latin *res* = 'thing'] to apply only to *things* and not to mere *states* or *qualities*; for example a thing's location and size and shape are not real because they are not themselves things but are spatial qualities of things. Now, the universal principle of the dynamics of material Nature is this:

> Everything that is real in the objects of our external senses must be regarded as a moving force.

This principle banishes from natural science the empty concept of the so-called solid, i.e. the concept of absolute impenetrability, and replaces it by the concept of repelling force. On the other hand, the true and unmediated attraction is •defended against all the bad arguments of a metaphysics that misunderstands itself, and •is explained as a fundamental force that is necessary for the very possibility of the concept of matter. One consequence of this is that we can if necessary think of space as filled throughout but in varying degrees, ·so that we can think of a portion of matter as light or soft or undense· without having to suppose that it has pockets of empty space scattered through it. To understand this, consider these two:

- **(1)** The basic repelling forces of matter, which are the basis for matter's first property, namely the filling of space;
- **(2)** The basic attraction of matter—the attraction that every portion of matter exerts on every other and also 524 the attraction that holds the portion together as a unit.

Now, **(1)** doesn't run in harness with **(2)**; on the contrary, we can think of their relationship to one another as infinitely diverse. This is because **(2)** rests on the amount [*Menge*] of matter in a given space, while **(1)** matter rests on the degree to which the space is filled—and this degree can vary enormously (as the same quantity [*Quantität*] of air in the same volume exhibits more or less elasticity according to its temperature). The underlying difference is this:

- **(2)** in true attraction all particles of matter act directly on all other particles of matter; whereas
- **(1)** by expansive force there is only action between the particles at the surface of contact between the two portions, and it makes no difference what the state of affairs is—whether there is much or little of this matter—behind this surface.

All this brings a *great* advantage to natural science, by relieving it of the burden of imagining a world built up out of full ·parts of space· and empty ones, allow it instead to think of all regions of space as full, but filled in varying measure [= 'in different degrees']. This at least deprives empty space of its status as *necessary*. It used to be thought of as *required* to explain differences in the weight or density etc. of different portions of matter, but now the thesis that there is absolutely empty space is reduced to the status of an *hypothesis*. [From here to the end of this chapter Kant will repeatedly contrast two different accounts of the fundamental nature of the physical world. To make it easier to keep the thread, the two will be given numerical labels within curly brackets, which aren't used for any other purpose in this document.]

[Kant begins his next paragraph by speaking of the advantage that {2} 'a methodically employed metaphysics' has over

{1} 'principles that are also metaphysical but haven't been subjected to the test of criticism'. That last word translates *Kritik*, which occurs in the German title of the *Critique of Pure Reason*. Its appearance here is sudden and surprising; it hasn't occurred earlier in this work except as part of that title; but Kant evidently expects us to gather that the difference between

{1} the common atomist metaphysic that deals in basic solidity, absolute impenetrability, and empty space

and

{2} his metaphysic of basic forces and degrees of intensity of fullness of space

is the difference between {1} a metaphysic that •hasn't been subjected to the kind of criticism that is central to the *Critique of Pure Reason* and {2} a metaphysic that •has. He says that the advantage of {2} over {1} is 'apparently only negative'. (Perhaps his thought is that {2} seems at first sight to do nothing but stop {1} from saying some of the things it is saying.) Anyway, {2} does in an indirect way enlarge the scope of the investigator of Nature, Kant continues:] because the conditions by which he previously limited his field, and by which all basic moving forces were philosophized away, now lose their validity, ·so that he has at his disposal some good concepts that he had thought were illegitimate; and *that* advantage is not 'only negative'·. But he—·this liberated investigator of Nature·—must be careful not to go beyond what makes the •universal concept of matter in general possible by trying to explain *a priori* any •specific facts about kinds of matter, let alone facts about •particular material things. The concept of matter is reduced to nothing but moving forces; that was to be expected, because in space the only activity, the only change, that is conceivable is motion. But who would claim to comprehend the possibility of fundamental forces? [See note on 'conceivable' on page 31.] They can only be *assumed*; ·and it is all right to assume them· if they inseparably belong to a concept that is provably basic and not further derivable from any other (such as the concept of the filling of space). These basic forces are the •repelling forces in general and the counteracting •attractive forces in general. We can quite well form *a priori* judgments concerning their inter-relations and consequences; the investigator is free to think up any relations he likes among these forces, provided he doesn't contradict himself. But he mustn't assume either of them as *actual*, because he is flatly not entitled to set up a hypothesis unless the *possibility* of what is assumed in it is entirely certain; and the possibility of the basic forces can never be comprehended ·and so can never be entirely certain·. And this points to an advantage that {1} the mathematico-mechanical kind of definition has over {2} the metaphysico-dynamical kind, namely: Starting 525
with

(a) a single completely homogeneous basic kind of material

—·namely absolutely solid matter·—this {1} mathematico-mechanical mode can provide for a great variety of sorts of matter that differ in density and (if it adds forces from outside the basic material) different modes of action. To do this, it needs the help of

(b) the different shapes that matter can have, and

(c) empty spaces between the portions of matter,

·But the addition of those two doesn't weaken the system in any way·, because the possibility of **(b)** the shapes and of **(c)** the empty intermediate spaces can be proved with mathematical evidentness. In contrast with this, if {2} the basic •material is transformed into basic •forces, then we don't have the means for constructing this concept or for presenting as possible in intuition what we thought universally. Why not? Because there's no secure way of explaining different sorts of matter in terms of different patterns of the basic forces; indeed, we can't even determine *a priori* what the laws are that govern those forces. But {1} a merely mathematical physics pays a high price for that advantage, because •it has to base itself on an empty concept (absolute impenetrability), and because •it must forgo all matter's own forces, ·and make do with forces from outside·. ·And in addition to those two defects, {1} also runs a risk·: Employing its basic patterns of portions of solid matter interspersed with empty spaces, it is required to provide explanations ·of the variety in sorts of matter·, and this requires it to allow—and to insist on its right to—a greater freedom of imagination than is prudent.

Starting with {2} basic forces I can't adequately show the possibility of matter or explain the different sorts of matter. But all that variety can be brought *a priori* under a few intermediate headings, and I do hope to present a complete account of those. (Not that this will provide a way of conceiving the possibility of matter.). . . . ·This material will be presented in four groups·.

(i) A body in the physical sense of the word is a portion of matter that has determinate boundaries and therefore has a shape. The size of the space within these boundaries is the body's **volume**. The degree to which a space is filled is called **density**. . . . The {1} system of absolute impenetrability provides for something to have *absolute* density, by having a portion of matter that has absolutely no empty spaces inside it. Using this concept of the filling of space, one portion of matter counts as less dense than another if it contains less empty space than the other, the extreme case being that of a portion of matter that is called *perfectly dense* because there is no empty space within its boundaries. The phrase 'perfectly dense' has a use in the context of {1} the merely mathematical concept of matter, and *only* there. In {2} the dynamical system, which has only *relative* impenetrability, there is no maximum or minimum of density. In that context any portion of matter can be called 'fully dense' if it has no empty spaces within its boundaries. i.e. if it is a continuum 526
and not an interruptum; and this implies nothing about how thin—·airy, light, etc.·—it is. And one portion of matter counts as 'less dense', in {2} the dynamical sense, than another if it entirely fills its space but not to the same degree as the other. But even in the dynamical system it's not satisfactory to make a 'density' comparison between two portions of matter unless they are homogeneous with one another, ·i.e. of the same kind·, so that one can be produced from the other by mere compression. Now, it doesn't appear to be essential to the nature of matter as such that any portion of it could be made indistinguishable from any other by compression, we shouldn't make density comparisons between heterogeneous portions or kinds of matter, as people customarily do when they say that water is less dense than mercury. [The {1}/{2} labelling will turn up again on page 46.]

(ii) Attraction when considered as acting between things that are in contact is called **cohesion**.

(It's true that some very good experiments have shown that the force that is called 'cohesion' when it operates between things that are in contact with one another is also active at a very small distance. But attraction across small distances is hardly perceivable; so when we speak of 'cohesion' we are thinking of things that are in contact. Cohesion is commonly taken to be a property that all matter has—not •derivable from the concept of matter but •shown by experience to be a feature of all matter. This universality mustn't be misunderstood as meaning **(a)** that every portion of matter is constantly exerting this kind of attraction on every other portion of matter —like gravitation—but rather as meaning **(b)** that every portion of matter acts in this way on any other portion of matter that comes into contact with it. [Kant describes these two versions of the force's universality as 'collective' (he could have said 'conjuctive') and 'disjunctive' respectively: in **(a)** a portion x acts in the relevant way on y **and** z and w and. . . etc, while in **(b)** it acts on y **or** w or z or. . . and so on, depending on which of these comes into contact with it.] For that reason, and also because this attraction is not a penetrating force but only a superficial one (there's plenty of evidence for that), its strength isn't always proportional to the density ·of the matter involved·. What is needed for two portions of matter to cohere with full strength is for them to be *first* fluid and *then* rigid; . . . . when a looking-glass has a crack across it, the portions of glass on the two sides of the crack are nowhere near to being as strongly attracted as they were when they became solid after being fluid. For all these reasons I regard this attraction-in-contact as only a derivative force of nature, not a fundamental one. But more of this later.)

A portion of matter is **fluid** if any moving force, however small, is sufficient to re-arrange its parts. The parts of a portion of matter are **re-arranged** when they are made to
527 switch places while remaining completely in contact with one another. Portions of matter—including the parts of a portion of matter—are **separated** if they lose all contact with one another or the amount of contact is lessened. A rigid body is one whose parts can't be re-arranged by any force—so these parts must be resisting re-arrangement by a certain degree of force ·of their own·. The resistance to the re-arrangement of portions of matter is **friction**. The resistance to the separation of portions of matter that are in contact is **cohesion**. So fluid portions of matter don't undergo friction when they divide; and where friction is found the portions of matter are assumed to be more or less rigid, at least in their smaller parts. . . . A rigid body is **brittle** if its parts can't be re-arranged without its breaking, so that the way its parts cohere can't be changed without its losing cohesion altogether. It is quite wrong to say (·as some do·) that •the difference between fluid and solid portions of matter comes from the •difference in the degree to which their parts cohere. When we call a portion of matter 'fluid', we aren't talking about •how resistant it is to being broken apart, but only about •how resistant it is to being re-arranged. Its •cohesion can be as strong as you like, but its •resistance to re-arrangement equals zero. Consider a drop of water. If a particle within the drop is drawn to one side by a very strong attraction of the parts touching it on that side, then it will be drawn just as strongly to the opposite side; and since the attractions cancel out, the particle is as easily movable as if it were in empty space. That's because any force that might move it has no cohesion to overcome; the only resistance to it would be the matter's so-called *inertia*, and that has to be overcome in making *any* matter move, even matter that doesn't cohere at all. Therefore, a microscopic bug will move as easily within this drop as if there were no cohesion to overcome. For in fact it *doesn't* have to lessen the water's cohesion—to pull particles of the water apart from one another—but only to re-arrange them. [Kant goes on to explain that if the bug tries to escape from the drop, then it does now have to overcome the water's cohesion, but not in a way that lessens the strength of the water's holding together as a cohering drop. He continues:] So it is clear that an increase of the cohesion of the parts of a portion of matter hasn't the slightest effect on its fluidity. Water coheres in its parts much more strongly than is commonly thought. . . . 528
What is quite decisive with regard to our concept of fluidity is this: fluid portions of matter can be defined as those in which the forces exerted by or acting upon each point are the same in every direction. The first law of hydrostatics is based

on the property of fluidity; and it can't be a property of an aggregation of smooth solid particles . . . . The considerations we are in among here enable us to show that fluidity is a *basic* property. ·If it were not basic but derivative, there would be portions of matter that were *very* but not *perfectly* fluid, and there aren't any·. If in a fluid portion of matter there was a tiny hindrance to re-arrangement and hence a tiny amount of friction, this friction would grow with the strength of the pressure with which the portion's parts are pressed against one another, and a strong enough pressure would have the effect that the parts of this portion of matter couldn't be re-arranged by any small force, ·i.e. the effect that the portion would no longer be fluid·. ·Here is a concrete example·:

> Take a U-shaped tube, of which one arm is very wide and the other very narrow (but not as narrow as a capillary, ·because that would have effects that would cloud our result·). Let both arms be a few hundred feet high. According to the laws of hydrostatics, the fluid in the narrow arm would reach exactly the height of the fluid in the wide arm (they are arms of a single tube). But now let us keep adding fluid to the tube, steadily increasing the pressure on the matter at the bottom of the tubes. It there were the tiniest potentiality for friction there, then at *some* height-of-fluid the movement of matter between the arms would stop: adding a small quantity of water to the narrower tube wouldn't affect the height of the water in the wider
> 529 tube; so that the narrow arm's column of fluid could be made to rise higher and higher above the wider arm's column.

And this is contrary to experience and even to the concept of fluidity. The same thing holds if, instead of unlimited pressure by weight, we postulate unlimited cohesion of the parts. I have presented two definitions of fluidity:

> **(a)** A portion of matter is **fluid** if any moving force, however small, is sufficient to re-arrange its parts.
> **(b) Fluid** portions of matter can be defined as those in which the forces exerted by or acting upon each point are the same in every direction.

We can derive **(b)** from the conjunction of **(a)** and the **(c)** principle of general dynamics saying that *all matter is basically elastic*, as follows: A portion of matter that is **(c)** elastic will resist, by stretching, any force of compression to which it is subjected; and if it is **(a)**-fluid, its force of recovery will equal the force of compression (nothing will be lost to friction); which is to say that the forces at work in it will be the same in every direction, i.e. that this portion of matter is **(b)**-fluid. So friction, properly so-called, can be had only by rigid portions of matter. . . . Some portions of matter that may have no more force of cohesion than some fluids nevertheless *strongly* resist the re-arrangement of their parts, so that they can't be pulled apart except by destroying the cohesion of all parts in a given surface, thus creating an illusion that they do have more cohesion. ·An example would be a cake of chocolate: you can *break* it in two, but you can't *pull* the two halves of it apart·. Such portions are *rigid* bodies. But why this is so, i.e. how rigid bodies are possible, is still an unsolved problem, though the ordinary doctrine of Nature [see note on page 1] thinks it has easily solved it.

(iii) A portion of matter may be able, after it has been deformed by an external force, to regain its original size and shape when the deforming force is removed; that ability is **elasticity**. When something can return to its previous size after being compressed, that is expansive elasticity; something that returns to its previous size after being stretched has attractive elasticity. The elasticity that consists only in the recovery of the previous shape is always attractive—e.g.

with a bent sword in which the parts on the convex surface have been pulled away from one another and try to resume their former closeness to one another. . . . (Attractive elasticity . . . . is obviously •derivative. An iron wire stretched by a weight springs back into its original size when the weight is removed. The attraction we have here is the cause of the cohesion of the wire. . . . Expansive elasticity, on the other
530 hand, may be either •basic or •derivative. Every portion of matter must, just because it is matter, have basic elasticity, but some also have derived elasticity. [Kant cites the example of hot air, which he thinks has—in addition to its basic elasticity—a further elasticity from being hot. He thinks that air's being hot is its being mixed with a special fluid, and that the elasticity of *this*—which may be basic—is passed on to or shared by the air. Finally:] It isn't always possible to know for sure whether a given instance of elasticity is basic or derived.

(iv) When moving bodies collide and alter one anothers' motion, that is called **mechanical** action. When a body is at rest [= 'motionless'] as a whole though its parts are moving around within it and interacting, their action is called **chemical**. When this chemical influence has the effect of pulling apart the parts of a portion of matter ·and inserting between them parts of another portion·, it is called **dissolving**. When the influence has the effect of separating out two portions of matter that have been dissolved in one another, it is called **chemical analysis**. . . . Absolute dissolving is the dissolving-in-one-another of two portions of matter of different kinds—·call them X and Y·—in such a way that *every* part of the X portion is united with a part of the Y portion in the same proportion as that of the solution as a whole. ·For example, if 5 units of X matter are merged into 10 units of Y matter, and this is 'absolute' dissolving, then in the resultant solution *every* part, however small, consists of one third X and two thirds Y.· This could also be called chemical penetration. (Whether the dissolving forces that actually occur in Nature are capable of bringing about a complete ·or absolute· dissolving doesn't have to be decided here. Here the question is only whether such a solution can be thought of.) Obviously if the parts of a dissolved portion of matter are still *particles*, it is as possible for •them to be dissolved as it was for •the larger parts to be dissolved; and if the dissolving force continues, it's not merely possible but inevitable that the dissolving will continue until *every* part of the solution is composed of X matter and Y matter in the same proportion as they have in the solution as a whole. Because in this case every part of the solution contains a part of the X matter, this matter must *completely fill* the volume in a continuous way. And the same holds for the Y matter. And when *each* of two portions of matter entirely fill a single space, they penetrate one another. That is why a perfect ·or absolute· chemical dissolving would involve penetration of the portions of matter. This chemical penetration would be entirely different from mechanical penetration. In the latter, the thought is that as ·two· portions of matter approach one another the repelling force of one could entirely outweigh that of the other, so that the extent of one or both of these portions of matter is reduced to nothing. In contrast with
that, in chemical penetration each portion of matter retains 531
its extent, but the portions are not outside one another but within one another. . . . It's hard to see any objection to the thesis that such perfect dissolving is possible, and thus that chemical penetration is possible. It does involve a completed division to infinity, ·and that seems to clash with the thesis that an infinite such-and-such is one that 'can't ever be wholly complete'· [see page 26], but:

(a) There there is no contradiction in *this* case of infinite division, because the dissolving takes place continuously throughout a period of time, i.e. through an infinite series of ever-shorter moments.

(b) Moreover, as the division proceeds the sums of the surfaces of the not-yet-divided portions of matter increase; and since the dissolving force acts continuously, the whole dissolving can be completed in a specifiable time.

If you think you can't conceive of such a chemical penetration of two portions of matter, that will be because *the divisibility to infinity of every continuum in general* really *is* inconceivable. If you won't accept this complete dissolving ·of one kind of matter in another· then you'll have to settle for an account that ends with certain small particles of the dissolved matter swimming around in the solvent at fixed distances from one another; these are still divisible portions of matter but ·according to you· they aren't also dissolved, and you won't be able to give the slightest explanation of *why*! It may be true in Nature, as far as our experience goes, that the solvent goes a certain distance and doesn't act further, ·but that is beside my present point·. My question concerns the *possibility* of a dissolving force that acts on smaller and smaller particles until the dissolving is completed. The volume of the resultant solution can be equal to the sum of the volumes of the two portions of matter before the mixture, or it can be smaller than this sum, or even larger than it, depending on how the attractive forces relate to the repelling forces, These mutually dissolving portions of matter constitute in solution, each of itself and both combined, an elastic medium. This elasticity provides the only sufficient reason why the dissolved X matter doesn't by its weight pull itself out from the Y solvent; it's because the solvent Y's attraction, since it occurs equally strongly toward all sides, destroys the resistance of the X dissolved matter. . . . [The next sentence and a half expands Kant's words in ways that the convention of ·small dots· can't easily indicate.] You might want to suggest that chemical dissolving will never be complete because the Y solvent will always be somewhat viscous, i.e. a bit thick and sticky. But that thought rests on the assumption that all dissolving consists in some X matter's coming apart and allowing some more fluid Y matter to flow between the parts; and this view of what dissolving is doesn't fit with the great force that the more solvent fluids exert on dissolved portions of matter—e.g. the action of dilute acids on metals. They don't merely touch the metallic bodies, which is what would happen if the particles of metal merely swam in the acid; rather, the acids exert great attractive force to pull these bodies apart and disperse them throughout the entire space of the containing flask. And another point: Even if •our knowledge and skills didn't put at our disposal any chemical forces of dissolving that could bring about a complete dissolving, •Nature might exhibit such forces in the operations of plants and animals, perhaps producing portions of matter that were products of complete or absolute dissolving though we had no way of separating the components out again. [Kant sketches 532
two possible examples of this, one involving heat and the other magnestism. They are hard to follow, and rest on now-exploded theories about those two phenomena. Then:] Our present search, though, is not for •hypotheses to explain particular phenomena but for •the principle according to which such hypotheses are all to be judged. Everything that frees us from the necessity of invoking empty spaces is an actual gain for natural science. Why? Because empty spaces leave the imagination far too free to invent fictions to make up for the lack of real knowledge of Nature. In the doctrine of Nature, absolute emptiness and absolute density play about the same role that blind chance and blind fate play in metaphysics, namely that of a bar to reason's dominance—either replacing it with fictions or lulling it to

sleep on the pillow of occult qualities!

The chief problem in natural science is to explain how there can be an infinite variety of kinds of matter. There are just two ways in which this can be attempted: {1} the **mechanical** way, by combinations of the absolutely full with the absolutely empty; and in opposition to that there is {2} a **dynamical** way in which all the varieties of matter are explained merely through combinations of the basic forces of repulsion and attraction. {1} The raw materials of the first are atoms and the void. An atom is a small portion of matter that is physically indivisible. A portion of matter is physically indivisible if its parts cohere with a force that can't be overcome by any existing moving force in Nature. An atom marked off from other atoms by its shape is called a primary particle. A body (or particle) whose moving force depends on its shape is called a machine. The **mechanical** natural philosophy is the process of explaining the variety of kinds of matter in terms of the nature and composition of their smallest parts, considered as machines. {2} And we can label as 'the **dynamical** natural philosophy' the explanation of the variety of kinds of matter not in terms of

- •particles considered as machines, i.e. as mere implements used by external moving forces,

but rather in terms of

- •the moving forces of attraction and repulsion that are inherent in these particles.

The {1} mechanical kind of explanation is very convenient for
533 mathematics, which is why it has—under the label 'atomism' or 'the corpuscular philosophy'—always maintained its authority over and influence on the principles of natural science, with little change from Democritus to Descartes and even to our own times. Its essentials consist in the assumption of

- •the absolute impenetrability of the basic matter,
- •the absolute homogeneity of this matter, with no differences except in shape, and
- •the absolute unconquerability of the cohesion of the matter in these basic particles.

Those were the materials for generating different kinds of matter in a manner that ·has two seemingly attractive features; it· •avails itself of a single basic kind of matter, varied only by the shapes of its portions; and •it explains Nature's various actions mechanically, as arising from the shape of these basic parts considered as machines that only needed an externally impressed force. But the claim of this system to be accepted depended, first and foremost, on the supposedly unavoidable necessity of explaining the different densities of kinds of matter in terms of *empty spaces*, which were assumed to be distributed among the particles and within each particle.... {2} A dynamical mode of explanation is far better suited to experimental philosophy [here = 'science'], because it leads directly to the discovery of •the moving forces that are inherent in portions of matter and •the laws of those forces, but doesn't freely allow the assumptions of empty intermediate spaces and fundamental particles with fixed shapes, neither of which can be discovered and determined by any experiment. To go to work in {2} this way we don't have to devise new hypotheses; all we need is to refute {1}'s postulate that it's impossible to think of different kinds of matter in any way except through the intermixture of portions of matter and empty spaces. And we *can* refute it, simply by showing how the different densities of kinds of matter can be consistently thought of without bringing in empty spaces.... This move rests on the fact that matter does not fill its space by *absolute* impenetrability but by repelling force; this force is a matter of *degree*, which can be
different in different portions of matter. The attractive force 534
of a portion of matter is proportional to the amount$_q$ of matter

in it, and is not correlated with its degree of repelling force; so the proportions of repelling to attractive force in different portions of matter can vary greatly. So there is no difficulty in the thought of a portion of matter that entirely fills its space without any empty parts and yet with only a tiny amount$_q$ of matter—so little that we can't detect it experimentally. This is one way to think about the ether. . . . The only reason for assuming an *ether* is to counter the claim that 'rarefied' matter can't be thought of except in terms of empty spaces. ·The outright assertion· that *there is ether* should not be made *a priori*, nor should any ·supposed· law about attractive or repelling forces. *Everything* must be concluded from data of experience—and that includes •the thesis that universal attraction is the cause of gravity, and •the laws of gravity. Still less will conclusions regarding chemical affinities be tested in any way except experimentally. Why? Because it lies right outside the scope of our reason to come at basic forces *a priori*. What natural philosophy [here = 'science'] does is to explain the variety of empirically encountered forces in terms of a smaller number of forces and powers; but these explanations go only as far as the fundamental forces—our reason can't get further down than that. Metaphysical investigation into the underpinnings of the empirical concept of matter is useful only for the purpose of leading natural philosophy as far as possible in the investigation of the dynamical grounds of explanation, because they provide our only hope of finding determinate laws and a system of explanations that hangs together in a rational way.

That is all that metaphysics can ever do for •the construction of the concept of matter, and thus on behalf of •the application of mathematics to the part of natural science dealing with •the properties by which a portion of matter fills a region of space in determinate measure. All metaphysics can do is to regard •these properties as dynamical and not as unconditioned basic *givens* such as a purely mathematical treatment would postulate.

I end this chapter with some remarks about the familiar question of the admissibility of empty spaces in the world. The possibility of such spaces can't be disputed. All forces of matter *presuppose* space; the laws governing the spread of these forces have the form 'If a region of space is. . . , then. . . ', so space is necessarily presupposed *before* all matter. Thus, attractive force is attributed to matter because matter occupies a space around itself by attraction, yet 525
without filling the space. So a region of space can be thought of as empty even when matter is active in it, so long as the activeness doesn't involve repelling forces, i.e. doesn't involve the matter's being *in* the space. But no •experience, •inference from experience, or •necessary hypothesis for explaining empty spaces can justify us in assuming that they are actual. Experience presents us only with *comparatively* empty spaces; and these can be perfectly explained in terms of the strength of the expansive force with which a portion of matter fills its space—·the *whole* of its space·—a strength that can be thought of as lesser and lesser to infinity, through all possible degrees, without requiring ·absolutely· empty spaces.

Chapter 3
Metaphysical Foundations of Mechanics

Definition 1

536 **Matter** is what can be moved, considered as having—just because it can be moved—a moving force.

Remark

This is the third definition of matter. The merely dynamical concept ·is different from this because it· applies also to matter that is motionless. The moving force that was in question back there concerned merely the *filling* of a certain region, and we weren't permitted to regard the matter that filled the space as itself moving. So repulsion was a basic moving force for imparting motion, whereas in mechanics a force is regarded as actually at work in one portion of matter imparting motion to another portion. ·Very briefly and schematically: the movement of portions of matter is considered as a •potential in dynamics, and as •actual in mechanics·. Clearly, a portion of matter won't have

> •the power to make other things move by its own motion

unless it has

> •*basic* moving forces through which it is active in every place where it exists,

this being an *activeness* that comes before any proper *motion*. ·Breaking that point down into its two constituents·: Clearly a portion of matter moving in a straight line and encountering another portion won't make the other move unless both of them have basic forces [Kant wrote *Gesetze* = 'laws', presumably a slip] of repulsion; and a portion of matter couldn't in moving drag another portion after it unless they both had attractive forces [*Kräfte* = 'forces']. So all mechanical laws presuppose
537 dynamical ones. . . . You'll notice that I shan't say anything more about the communication of motion by •attraction (such as might happen if a comet with a stronger attractive power than the earth came close to the earth and dragged it out of its orbit). I'll be talking only about the agency of •repelling forces—i.e. agency by pressure (as by means of tensed springs) or by impact. I'll do this because applying the laws of repulsion is exactly the same as applying the laws of attraction except for the difference in direction.

Definition 2

The **amount$_q$ of matter** there is in a certain space is given by how many movable ·parts· there are there. When this matter is thought of as having all its parts in motion at once, is called the mass; when all the parts of a portion of matter move in the same direction, exercising their moving force externally, the portion is said to **act in mass**. A mass with a determinate shape is called a **body** (in the mechanical sense). Mechanically estimated, •the amount of motion is ·a function of two variables, namely· •how much$_q$ matter is moved and •how fast. . . .

Proposition 1

The only way of comparing the amounts$_q$ of *any* two portions of matter is by comparing their amounts$_q$ of motion at a single speed.

Proof

Matter is infinitely divisible, so the amount$_q$ of matter in a given portion can't be determined directly by how many
parts it has. How much$_q$ matter there is in one portion of 538
matter can be directly compared with how much$_q$ there is in another, if the two are *of exactly the same kind*, because in that case the amounts$_q$ are proportional to the volumes. But

Proposition 1 concerns quantitative comparisons between *any* two portions of matter, including ones of different kinds. So there is no all-purpose method—direct or indirect— for comparing *any* two portions of matter with one another, if we ignore their motions. If we bring motion into the story we do get a universally valid procedure for such comparisons—and it is the only one we can have. It involves measuring the amounts of matter in terms of the amounts$_q$ of motion. But this comparison gives us what we want only if the two portions of matter are going at the same speed. Therefore etc.

Note

How much$_q$ motion a body has is how much$_q$ matter it has and how fast$_q$ it is moving. One body has twice the motion of another body if

- •they have the same speed, and one has twice as much$_q$ matter as the other, or
- •they have the same mass and one has twice the speed of the other.

That is because the determinate concept of a *size* or *amount* is possible only through the construction of the quantum; and such a construction involves putting together many items that are *equivalent* to one another [see 'wwhat Kant aseems to have meant' on page 14]. Thus, the construction of a motion's amount$_q$ is the putting together of many equivalent motions. Now, in the context of phoronomy there is no difference between

- •giving to a movable thing a speed S, and
- •giving to each of n equivalent movable things a speed of S/n.

The first thing we get from this is an *apparently* phoronomic concept of the amount$_q$ of a motion, as composed of many motions that are external to one another but constitute a single united whole. And if we think of each point as getting its •moving force from •how it is moving, this turns into a mechanical concept of the amount$_q$ of the motion. But ·actually this is a blind alley; we can't get at a mechanical concept of amount-of-motion in this way, because· in phoronomy we *can't* represent a motion as composed of many motions existing externally to one another. Why not? Because in phoronomy movable items are represented as mere points, with no moving force, so that the only basis for distinguishing the amounts$_q$ of motion of two things is in terms of their ·differences of· speed. [In a spectacularly obscure passage,
Kant goes on from there to compare measuring amounts of 539
•motion with measuring amounts of •action, and to criticise a wrong idea that some theorists have had about the latter. His purpose in going into all this seems to be to present some thoughts about differentiating 'dead forces' from 'living forces'. We hear no more of that distinction in the present work, and Kant invites us to bypass it when he ends by saying:] . . . .if indeed the terminology of 'dead force' and 'living force' deserves to be retained at all.

Remark

I have things to say in explanation of the preceding three statements—[i.e. Definition 2, Proposition 1, and the following Note]—and in the interests of concentration I shall condense them into a single treatment.

Definition 2 says that the quantity [*Quantität*] of a portion of matter can only be thought of in terms of how many movable parts (external to one another) it has. This is a remarkable and fundamental statement of universal mechanics, ·because it supplies an answer to the important question 'Can we have a concept of the *intensive* magnitude of an instance of moving force? The answer is that we cannot·. Such an intensive magnitude would have to be independent of •the amount of matter and of •the speed, both of which

are extensive magnitudes; and Definition 2 tells us that those are the only quantitative notions that are applicable to a portion of matter. Intensive magnitude *would* have a place if matter consisted of monads. ·A monad has (by definition) no parts; so· any monad—however it was related to anything else—could be more or less real in some way that didn't depend on how many parts-external-to-one-another
540 it had, ·which means that its reality could be an *intensive* magnitude·. As for the concept of •mass in Definition 2: it is usually equated with the concept of •quantity, but this is wrong. Fluid portions of matter can by their own motion *act in mass* [see Definition 2] but they can also *act in flow*. In the so-called water-hammer—·which causes a knocking sound in the pipes when a flow of water is suddenly blocked·—the water in striking acts in mass, i.e. with all its parts simultaneously; the same is true when a pot full of water is weighed on a scale. But when the water of a millstream acts on the lower paddles of the wheel, it doesn't do so in mass, i.e. with all its parts *together* colliding with the wheel; rather, the parts act successively. So if in this case we want to determine how much$_q$ matter is being moved with a certain speed and exerting moving force, we must first of all look for *the body of water*, i.e. find out how much$_q$ matter can produce the same action when it acts in mass (by bringing its weight to bear) with a certain speed. That's why we usually understand by the word 'mass' the amount$_q$ of a *solid* body (a fluid can be treated as solid on the strength of the vessel containing it). Finally, there's something odd about the Proposition and its appended Note. According to the Proposition,

•how much$_q$ matter must be estimated by how much$_q$ motion at a given speed,

whereas according to the Note,

•how much$_q$ motion. . . .must be estimated by how much$_q$ moved matter.

This seems to revolve in a circle, offering no prospect of a determinate concept of either of the terms. It really would be circular if these were definitions of concepts in terms of one another, but that's not what is going on. The Proposition does define a concept, but all the Note does is to explain how that concept applies to experience. . . .

This should be noted: For any given portion of matter, the question 'How much [*Quantität*] matter is there in this?' is the question 'How much [*Quantität*] **substance** is there in this?' and *not* 'How much [*Größe*] of **quality** Q is there in this?', where Q is some special quality such as the powers of repulsion or attraction that are cited in dynamics. And what is meant in this context by 'the amount [*Quantum*] of substance' is merely 'how many movable parts' there are in the given portion of matter.

[Throughout this paragraph, 'how many' translates the German noun *Menge*. Of the two other currently available English translations of this work, one says

(1) 'the mere aggregate of the movable'

while the other says

(2) 'the mere number of the movable parts'.

Of these, the second is not quite right, but is nearer to right than the other. As you might guess, *Menge* has two meanings. **(1)** It can be a concrete noun, meaning something like 'multitude' or 'crowd' or, if you like, 'aggregate'. 'I looked along the street and saw a *Menge* of angry people coming towards me'. **(2)** It also has a sense in which it is an abstract noun, meaning something like 'how-many-ness'. Why say it in that clumsy way, rather than just using 'number' as the translator did? Because Kant sometimes—notably in the *Critique of Pure Reason*—uses *Menge* as his more general how-many concept while reserving *Zahl* = 'number' to mean '*Menge* that is determinate'. He holds that when there are infinitely many Fs, the *Menge* of Fs is not determinate, and so there is no such thing as the *Zahl* of Fs; the phrase 'infinite number' is, he holds, self-contradictory. In the present work, most occurrences of *Menge* are in the context of items of which there are infinitely or endlessly many, so that Kant couldn't have used *Zahl* (which in fact occurs only twice in the whole work). The more important point, however, is that all through this work *Menge* is being used as an abstract 'how-many' noun and not as a concrete noun meaning 'crowd' or the like.]

[What follows expands and re-arranges what Kant wrote, in ways that the usual conventions of ·small dots· can't easily indicate. The thoughts expressed here are all present, explicitly or implicitly, in the paragraph that is being replaced.] This emphasis on how-many-parts is justified by a deep theoretical point about the line between *substance* and *quality*. The concept of substance is the concept of the rock-bottom subject, i.e. a subject that isn't in its turn a quality of another subject. Now, we want to get a grip on a notion of *how much* substance there is in a given portion of matter; and we can't get at it through any such notion as that of how-much-force the portion has or how-big the portion is, because force and size belong on the
541 quality side of the fundamental substance/quality line. Well, then, what *isn't* on that side of the line? The only candidate is how-many-ness: 'How many movable parts of substance S are there?' isn't a question about any of S's qualities, so the answer to it doesn't slide across to the wrong side of the line. [The remainder of the 'Remark' is omitted because the preparer to this version has been defeated by it. On page 67 the passage is presented in each of the two currently available English translations of it.]

Proposition 2

First law of mechanics: Through all changes of corporeal Nature, the over-all amount$_q$ of matter remains the same—neither increased nor lessened.

Proof

(Universal metaphysics contains the proposition that through all changes of Nature no substance either comes into existence or goes out of existence; all that mechanics is adding here is an account of what substance in matter *is*.) In every portion of matter the movable in space is the ultimate subject of all qualities that matter has, and •how many movable parts
external to one another a portion of matter has is •how much 542
substance there is to it. Hence the amount of any portion of material substance is nothing but how many substances it consists of. So the only way the amount$_q$ of matter could be increased or lessened would be for material substances to go out of existence or for new ones to come into existence. But substances *never* come into or go out of existence in changes of matter. So the over-all amount$_q$ of matter in the world is neither increased nor lessened in these changes, but remains always the same.

Remark

The essential thing about substance as it figures in this proof—only as existing in space and subject to the conditions of space, and hence as having to be an object of the outer senses—is that the amount of it can't be increased or diminished unless ·some· substance comes into or goes out of existence. Why not? Because if x is something that can exist only in space, the amount of x that there is has to consist in ·facts about· the parts that x has external to one another, and if these are real (i.e. are movable) they must necessarily be substances. On the other hand, something regarded as an object of inner sense can as substance have an amount or magnitude that doesn't consist of parts external to one another, so that the parts that it does have are not substances. When this item comes into or goes out of existence, that doesn't involve any *substance's* doing so; so the magnitude of the item can increase or lessen without detriment to the principle of the permanence of *substance*. [On this and the next two pages Kant uses the phrase 'the permanence (*Beharrlichkeit*) of substance' to mean 'the fact that no substance comes

into, or goes out of, existence'. On pages 11–12 the cognate adjective, *beharrlich*, being used for a quite different purpose, was translated as 'time-taking'.] How can that happen? Well, I can be more conscious or less conscious, so my mental representations can be clearer or less clear, and this gives to my faculty of consciousness—I call it 'Self-awareness'—a degree ·of reality·, and we can even say that the substance of my soul has such a degree; and none of this in any way requires that any substance come into existence or go out of existence. This faculty of Self-awareness can gradually diminish, to the point where it finally goes right out of existence, so the substance of the soul can gradually go out of existence. [In this sentence and the preceding one, Kant doesn't say that the soul is a substance; he speaks of the 'substance *of* the soul'. He doesn't explain the 'substance *of*' locution (which occurs nowhere else in this work, and nowhere in the *Critique of Pure Reason*). It does save him from having contradicted himself about whether substances can go out of existence.] If a thing has parts external to one another, the only way it could go out of existence gradually is by being slowly dismantled, pulled apart; but the soul can go out of existence gradually in a different way, through being gradually lessened and eventually extinguished. [Kant's next sentence is hard to follow. In it he sketches, in a condensed form, some doctrine from the *Critique of Pure Reason*. He is facing the challenge 'Don't we know that the soul is a substance? Isn't it obvious that when I say "I see something red" I am attributing the predicate "sees something red" to the mental thing, the substance, that I call "I"?' Kant rejects this, and gestures towards the *Critique*'s account of how 'I' *does* work in all its uses. Fortunately, we don't really need that account for present purposes. All that matters here (and even it doesn't here matter *much*) is his negative thesis that 'I' or the German *Ich* does not serve to pick out an individual thing, and therefore isn't the name of a substance. Kant winds this up by saying that the person who uses 'I' isn't employing any concept of himself as a substance, and he is clearly implying that there is no such concept. Then:] In contrast 543
with that, the concept of a portion of matter as substance is the concept of *something that is movable in space*. So it's not surprising that the permanence of substance can be proved of matter but not of the soul. This is because from the concept of matter as *what is movable in space* it follows that the quantitative or *how-much* aspect of matter depends on there being many •real parts external to one another—and thus many •substances. Thus, the going out of existence of a portion of matter ·would involve the going out of existence of many substances, and that· is impossible according to the law of permanence. [Kant has *Gesetz der Stetigkeit* = 'law of continuity' here, an obvious slip.] (The portion of matter could be diminished by being taken apart, but that isn't the same as going out of existence.) The thought 'I', on the other hand, isn't a *concept* at all but only an inner perception. And nothing *follows from* this thought (except that an object of inner sense is completely distinct from anything that is thought of merely as an object of outer sense); so the permanence of the soul as substance doesn't follow from it.

Proposition 3

Second law of mechanics: Every change in matter has an external cause. (Every motionless body remains at rest, and every moving body continues to move in the same direction at the same speed, unless an external cause compels it to change.)

Proof

(Universal metaphysics contains the proposition that every change has a cause. All we have to do here ·in mechanics· is to prove with regard to matter that every change in it must have an *external* cause.) Because matter is a mere object of outer sense, the only facts about it are facts about how portions of matter relate to one another in space; and from this it follows that the only way there can be any change in matter is through motion—i.e. through •changes from motion to rest or vice versa, or •changes in direction and speed of motion. The principle of metaphysics says that each such change must have a cause; and this cause can't be internal, because matter has no absolutely inner states or inner causal resources. Hence all change of a portion of matter is based on an *external* cause.

Remark

The name 'law of inertia' should be given only to •this law
544 of mechanics, and not to •the law that every action has an equal and opposite reaction. The latter says what matter *does*, but the former says only what it *doesn't do*, and that is a better fit for the word 'inertia'. To say that matter 'has inertia' is just to say that matter in itself is *lifeless*. For a substance to have *life* is for it to be able to get itself, through its own inner resources, to *act*—i.e. to change in some way (for any finite substance) or start or stop moving (for any material substance). Now, the only inner resource we know of through which a substance might change its state is *desire*, along with its dependents—•feelings of pleasure and unpleasure, •appetite, and •will—and the only inner activity that we know of is *thought*. But none of these causes and activities have anything to do with the representations of outer sense, and so they don't belong to matter as matter. Therefore all matter as such is lifeless; and that is what Proposition 3, the one about inertia, says—and it's *all* it says. If we want to explain any change in a material thing in terms of *life*, we'll have to look for this cause in some other substance that is different from matter although bound up with it. That's because in gathering knowledge about Nature we must •first discover the laws of matter as such, not mixing them up with any other active causes, and •then connect these laws with any other causes there may be, in order to get a clear view of exactly what each law of matter brings about unaided. The possibility of a natural science proper rests entirely on the law of inertia (along with the law of the permanence of substance). Hylozoism [= 'the thesis that matter itself is alive'] is the opposite of this, and is therefore the death of all natural philosophy! Just from the concept of inertia as •lifelessness we can infer that 'inertia' doesn't signify a thing's •positive effort to maintain its state. Only living things can be called 'inert' in this positive sense; it involves their having a thought of another state that they don't want to be in and do their best to avoid.

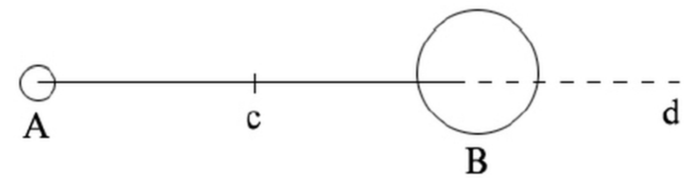
Proposition 4

Third mechanical law: In all communication of motion, action and reaction are always equal to one another.

Proof

[In a notably obscure explanation—omitted here—of why he had to deal with this third law, 'for the sake of completeness', Kant refers to it as 'the law of two-way causal interaction of universal metaphysics'. His word
for 'two-way causal interaction' is *Gemeinschaft*, which is standardly but 545
unhelpfully translated as 'community'.] Active relations of portions of matter in space, and changes of these relations, have to be represented as reciprocal if they are to be ·thought of as· causes of certain effects. Now, any change of such relations is *motion*; so we get the result that whenever one body causes a change in another body, the other must also be in motion (so that the interaction can go both ways); so we can't allow for any case in which one body A causes motion in another body B which until that moment was •absolutely

at rest. What we can do is to represent B as being at rest •relative to the space to which it is referred; B must be represented as moving with its reference-space towards A, moving at the same speed in absolute space as A is moving towards B. For the change of relation (and hence the motion) is completely reciprocal between both bodies; by as much as A approaches every part of B, by that much B approaches every part of A. What we are dealing with here is not the empirical space surrounding the two bodies but only the line stretching between them (because our whole topic is just the effect that the movement of each has on the state of the other, and for that we can abstract from all relation to empirical space); and therefore we think about their motion only in terms of absolute space, in which they share equally in the motion attributed to A, the one in relative space, because there's no basis for attributing more motion to A than to B. On this footing, the motion of a body A toward an immobile body B is handled in terms of absolute space, i.e. the motion in question is treated as a relation of two causes interacting with one another and not with anything else; and so the motion which appears to us as only A's is considered as something shared between A and B. This can occur only in the following way. The speed which in the relative space is attributed only to A is divided between A and B in inverse proportion to their masses; A's share is only its speed in absolute space, whereas B (*along with the relative space in which it is at rest*) is assigned its speed in the opposite direction; and in this way the same appearance—·i.e. the appearance that A moves towards B, which is motionless·—is perfectly retained. [We are about to see Kant representing speeds by lines, in accordance with his statement on page 11 that 'In phoronomy we use the word "speed" with a merely spatial meaning—the measure of how far a thing travels in a given period of time'—which has the result: the longer the line, the faster the motion.] What happens in the two-way causal interaction of the bodies is constructed as
follows. 546



Let a body A be moving into a collision with the body B with a speed = AB with regard to the relative space in relation to which the body B is at rest. Let the speed AB be divided into two parts, Ac and Bc, in such a way that their respective speeds are inversely proportional to their respective masses. Represent A as moved with the speed Ac in absolute space, and ·the larger body· B (*together with the relative space*) as moving with the ·smaller· speed Bc in the opposite direction. Thus the two motions are opposite and equal to one another. [Kant is relying here on the thesis (page 49) that in mechanics the concept of *how much motion* is a function of speed *and mass*.] ·Because they are equal and opposite, neither is the winner, and· they destroy one another and both come to be, relatively to one another, i.e. in absolute space, in a state of rest. [In a helpful footnote in his translation of this work (Cambridge University Press 2004), Michael Friedman points out that Kant is here discussing the collision of perfectly inelastic bodies, i.e. ones that have no bounce-back from a collision.] So we have B moving *with its relative space* in the BA direction, and losing its motion when it collides with A; but the collision doesn't automatically cancel the motion of B's relative space as well. So we have two equivalent ways of describing the state of affairs after the collision:

- •The bodies A and B are now at rest in absolute space, and relative to them the relative space moves in the direction BA with the speed Bc.

•The bodies A and B move with equal speed Bd = Bc in the direction AB, i.e. the direction that the impacting body A had.

Now, according to this,

the amount$_q$ of motion of B in the direction and with the speed Bc,

which is the same as

the amount$_q$ of motion of B in the direction Bd with the speed Bd,

is equal to

the amount$_q$ of motion of the body A with the speed and in the direction Ac.

Consequently, the effect ·of the collision·, i.e. the motion Bd that B receives by impact in relative space, and hence also the action of the body A with the speed Ac, is always equal to the reaction Bc. That is a part of what was to be proved—the part saying that whenever a moving body causes a change in the motion of a stationary body, action and reaction are equal. It's a thesis in mathematical mechanics that this same law ·about action and reaction· holds for the impact of one moving body on another *moving* body just as well as it does for the impact of one body on another that is *motiononless*. Also, the communicating of motion by *impact* differs from the communicating of motion by *traction* only in the direction in which the portions of matter oppose one
547 another in their motions. From all of this it follows that in all communication of motion, action and reaction are always equal to one another: an impact can communicate the motion of one body to another only by means of an equal counter-impact, a pressure only by means of an equal counter-pressure, and a traction only by an equal counter-traction).[3]

Note 1

From this we can infer a natural law that is of some im- 548
portance for universal mechanics, namely that *every* body, however great its mass may be, must be movable by the impact of *any* other body, however small its mass or speed may be. This is because the motion of body A in the direction AB *must* encounter an equal motion of B in the opposite direction BA [this refers to the diagram on page 54]. The two motions cancel one another in absolute space by impact. But thereby both bodies receive a speed Bd = Bc in the direction of the impacting one; consequently, the body B is

[3] In **phoronomy** a body's motion was considered merely as the change in its relation to space, so we could say that 'the body moves in *this* direction' or 'the body stays still and the space moves in *that* direction', i.e. the opposite direction; these two were indistinguishable by any appearance. The quantity of motion of the space was merely its speed, so the quantity of motion of the body was nothing but *its* speed (which is why we could regard the body as a mere movable point). But in **mechanics** we deal with a body x that is moving relative to another body y and, through that motion, is *causally* related to y. Whether by •moving towards y and exercising its force of impenetrability or by •moving away from y and exercising its force of attraction, x comes to be in a two-way causal interaction with y. So here ·in mechanics· there *is* a difference between 'x moves this way' and 'x is stationary while the space containing it moves the opposite way'. That is because we are now working with a different concept of *quantity of motion*—it involves not merely a thing's speed but also the thing's quantity of substance, which is relevant to its role as a moving cause. And we now •have to assume that both bodies are moved (in phoronomy we had our •choice about that), and indeed that they are moved with the same quantity of motion in opposite directions. When one body is not moving in relation to its space, we have to attribute the required motion to this body together with its space. For the only way the body x can act on body y other through x's s own motion is by repulsive force as it approaches y or by attractive force as it moves away. Now, given that the two forces always act with equal strength and in opposite directions, any action by one of those forces in one body requires a counter-action by the other; so no body can pass motion on to an absolutely immobile body; the second body *must* be moved in the opposite direction . . . .

movable by every force of impact, however small.

Note 2

This is, then, the **mechanical** law of the equality of action and reaction. It is based on the fact that motion is never communicated ·from one body to another· except in a two-way causal interaction between the two. And on the following: It is trivially obvious that a body A can't hit a second body B that is *motionless in relation to A*; what happens is that A hits B which is *motionless in relation to B's space*. So when A hits B, it must be that B *together with its space* is moving towards A. How fast? Well, the initial speed that we would attribute to A if we thought of it as moving in absolute space has to be divided between A and B-with-its-space, ·so that we get the right account of how long it takes A to reach B from the given distance away·; and the division must assign to each body not the same speed but the same amount of motion, so that the speed of each is inversely proportional to its mass. [See the discussion of amount-of-motion on page 49.]. . . .

There is also a **dynamical** law of the equality of the action and reaction of portions of matter. It doesn't concern one portion A's sharing its motion with another portion B, but rather A's giving its entirer motion to B and having motion produced in itself through B's resistance. This can be easily demonstrated in a similar way. For if A attracts B, then A compels B to approach A, i.e. resists the force with which B tries to pull away. But there's no difference between •B's pulling away from A and •A's pulling away from B; so traction and countertraction are equal to one another.
549 Similarly, if A repels B, then A resists the approach of B; but B's approaching A is just the same as A's approaching B, so it is just as correct to say that B equally resists the approach of A; so pressure and counterpressure are always equal.

Remark 1

This then is the construction of the communication of motion. This construction necessarily carries with it the law of the equality of action and reaction. Newton didn't venture to prove this law *a priori*, but appealed to experience to prove it. Others tried to secure this law by introducing into natural science a special force of 'inertia' (Kepler's name for it); so basically they were also deriving it—·i.e. the law of action and reaction·—from experience. Yet others tried to get it from the mere concept of *the communication of motion* [see the note on page 38]. They thought of this as involving a gradual transfer of body A's motion into another body B, so that A loses exactly as much as B gains; ·their view was that· the transfer stops when A and B are moving at the same speed in the same direction as that of the latter; and that rules out any reaction, i.e. any reacting force of B acting back against A that collides with it.[4] ·That is bad enough, but there is

[4] ·This theory didn't have to say that the motion-transfer is gradual·. The equality of A's action with B's. . . .is secured just as well if the transfer of motion is supposed to be instantaneous, with body A coming to rest immediately after the collision; and that's the form these theorists' account would *have to* take if they were thinking of the two bodies not as elastic but as absolutely hard. But the law of motion that *that* left them with doesn't square with experience and isn't even consistent, so their only way out would be to deny that there are any absolutely hard bodies (thus making their law *contingent*, because dependent on a special quality of colliding bodies). But I can't see how the transfer-of-motion theorists could explain what happens in collisions if the colliding bodies are elastic. It is clear that when elastic body A collides with immobile elastic body B, it is *not* the case that B merely receives motion that A loses; rather, B exercises actual force in the opposite direction against A, as though it were pushing against a spring lying between them; and for this it requires just as much actual motion (but in the opposite direction) as A needs for its part in this transaction. In my version of this law, on the other hand, ·no such difficulty arises, because· it doesn't make the slightest difference whether the colliding bodies are absolutely hard

550 worse·. They don't show what their account *means*, i.e. they haven't explained 'communication of motion' in a way that shows that such communication is possible. [In the original, this 'Remark' down to here is a single sentence.] The mere words 'transfer of motion from one body to another' don't explain anything. If they are understood literally, implying that motion is poured from one body into another like pouring water from one glass into another, they conflict with the principle that *qualities don't wander from one substance to another* [Kant gives this in Latin]. If that literal reading is rejected, then the theorists I am discussing must face the problem of how to make this possibility conceivable. . . . The only way to make sense of A's motion's being necessarily connected with B's motion is by attributing to both bodies dynamic forces (e.g. the force of repulsion) that precede all motion—·i.e. so that the forces explain the motions, not vice versa·. And *then* we can prove that the motion of A towards B is necessarily connected with the approach of B toward A; and if B is regarded as immobile, then A's motion is connected with the motion of B-together-with-its-space towards A, so that the bodies with their (basic) moving forces are considered to be moving relatively to one another. We can fully grasp this course of events *a priori*: whether or not the body B is moving in relation to its empirically knowable space, we have to regard it as moving in relation to the body A, and indeed moving in the opposite direction. Otherwise, A's movement couldn't bring into action the repelling forces of itself and of B, in which case portions of matter couldn't act mechanically on one another in any way, i.e. there couldn't be any communication of motion through collisions.

Remark 2

The name 'force of inertia' must be dismissed from natural science (despite the fame of its inventor). This must be done •because the phrase is inherently self-contradictory; and because because the ·so-called· 'law of *inertia*' (law of lifelessness!) could easily be confused with the law of *reaction*; and above all •because this confusion would support and encourage the wrong account given by those who don't have a proper grasp of the mechanical laws. According to *their* account, the reactions of bodies—now described as ·exercises of· 'the force of inertia'—would lead to

> **(i)** the lessening or annihilation of all the motion in the world,

and to

> **(ii)** collisions in which no motion is communicated.

The reason for **(i)** is this: according to the account that I am attacking, the moving body A would have to 'spend' some of its motion in overcoming the inertia of the immobile body B, and that 'expense' would be sheer loss. And the reason for **(ii)** is this: If B were very massive ·and A much less so·, A wouldn't have enough motion *both* to overcome B's 'inertia' *and* then to make B move; so that this would be a collision in which no motion was communicated. ·Summing up·: 551
A motion can't be resisted by anything except an opposite motion; it can't be resisted by a body's immobility! So the 'inertia' of matter, i.e. its mere incapacity to get itself moving, isn't the cause of any resistance. It could be defined:

> 'inertia' = 'a unique force to resist a body but not to move it'

and that would make 'inertia', ·despite its definition·, a word without any meaning. We could put 'inertia' to a better use by designating the three laws of universal mechanics as:

or not. [Kant is referring to his version of the 'third mechanical law'—Proposition 4 on page 53. So his topic in this footnote, where he writes about 'the law' that these transfer-of-motion theorists have to accept, is really their *version of*, their *presentation of*, the third mechanical law.]

•the law of matter's subsistence [Proposition 2, page 51],
•the law of matter's inertia [Proposition 3, page 52], and
•the law of the reaction of portions of matter [Proposition 4, page 53].

These laws, and hence all the propositions of mechanical science, correspond exactly to the categories of •substance, •causality, and •two-way interaction. There is no need for me to discuss this here.

GENERAL REMARK ON MECHANICS

The communication of motion takes place only by means of moving forces—impenetrability and attraction—that a portion of matter also has when it is not moving. The action of a moving force on a body at an instant is the solicitation of the body. [That's the first appearance of 'solicitation' in this work. It is or was a technical term in mechanics. You can safely think of it as meaning 'instantaneous tug or push'.] The speed of the body brought about by its solicitation—understood in terms of how this speed can increase uniformly through time—is the acceleration-at-a-moment value. (The latter must involve only an infinitely small speed, because otherwise the body would attain through the acceleration value an infinite speed in a given time, which is impossible. . . .) As an example of the solicitation of matter by expansive force, let us consider compressed air holding up a weight. In this situation, the air's exercise of expansive force must have a finite speed. [In this paragraph, 'finite' always means 'more than infinitely small'.] Why? Because expansive force occurs only at the surface, which means that it is the motion of an infinitely small amount of matter; and so we have on the air's side of the transaction

•an infinitely small amount of matter with a finite speed,

which has to balance, or to equal, what there is on the weight's side, namely

•a body of finite mass with an infinitely small speed.

Whereas expansive force operates only at the surface, and is therefore a force exercised by an infinitely small amount of matter, attraction is a penetrating force: a body's attractive force penetrates the body itself, so that the body's inner parts contribute to the attractive force of the body as a whole. If attraction were not a penetrating force, the equations implied by the mechanical proposition 4 wouldn't come out right. [That somewhat simplifies what Kant wrote.] *Cohesion* is 552
often thought of as a force operating only at surfaces; but we now see that if cohesion is to be true attraction and not merely external compression—i.e. if it's to be thought of in terms of the parts of a body *pulling* together rather than being *pushed* together—it can't be thought of in this way.

An absolutely hard body would be one whose parts attracted one another so strongly that no weight could •separate or •re-arrange them. This means that the parts of such a body would have to pull on one another infinitely more strongly than gravity pulls on them (·because: *however* strong the gravitational pull, the part-on-part pull will defeat it·). But . . . [and then Kant proceeds with a defeatingly technical reason why this fact, conjoined with some others, implies that absolute hardness is impossible. He follows this with what seems to be an entirely different and much more accessible reason, namely:] An absolutely hard body is impossible because

> in a collision between body x and absolutely hard body y, x would be moving with a finite speed and y would react *instantaneously* with a resistance equal to the whole of x's force.

And this is impossible. A portion of matter produces by its impenetrability or cohesion only an infinitely small instantaneous resistance to the force of a body that collides with it. A consequence of this is the mechanical law of continuity, namely:

> A collision can't make a body move, or stop moving, or change speed or direction, *instanteously*. Any such change occurs through a time-taking infinite series of intermediate states whose difference from one another is smaller than that between the first and last such states.

[That is what Kant wrote; but the reference to 'the first and last' states doesn't help to pin down the notion of continuity, and may be a mere slip on Kant's part. What he needed, and perhaps what he meant, was this: 'Given *any* two members of this series, there is an intermediate state that is more like each of them than they are like one another.']

Thus, a moving body x that collides with a portion of matter y is halted by y's resistance not •instanteously but through a continuous slowing down; similarly for the other changes that a collision can subject a body to—starting to move, changing speed, changing direction. When the direction of a body's motion changes, it goes through all possible directions intermediate between its first and last ones, which means that it changes direction by moving in a curved line. This law ·of continuity· also applies. . . .to changes in a body's
553 state by means of attraction. This ·mechanical· law of continuity is based on the law of the inertia of matter. On the other hand, the *metaphysical* law of continuity applies quite generally to all change (internal as well as external), and its basis is provided by concepts: the concept of *change* as a magnitude, and the concept of *generation of a magnitude* (which necessarily happens continuously through a period of time). So this metaphysical law has no place here.

Chapter 4 Metaphysical Foundations of Phenomenology

Definition

Matter is whatever is movable and can be an object of 554
experience.

Remark

Like everything that is represented through the senses, motion is given only as appearance. For the representation of motion to become experience, there has to be—in addition ·to **(i)** what is received through the senses·—also **(ii)** something thought by the understanding. As well as **(i)** being a state of the perceiving •subject, the representation ·of motion· must also determine an •object. So something movable becomes an object of experience when a certain object (here, a material thing) is thought of as falling under the predicate 'moves'. Now, motion is change of relation in space. So there are always two correlates here, ·namely matter and space, and we have some options·:

- **(a)** In appearance, we can handle things in terms of the motion of matter or the motion of space; it doesn't matter which we choose, because the two accounts are equivalent.
- **(b)** In the experience of motion we must think of one of the two—matter or space—as moving and the other as staying still.
- **(c)** Reason must necessarily represent both of these correlates as moving at the same time.

All we get in the appearance of motion is the change in
the relation of matter to space; and that doesn't pick out 555
any of those three options as the right one. But ·we can't

just leave it at that·: we have to settle the conditions under which a movable thing can be thought of as moving in this or that specific way, because without that there can't be *experience* of a moving thing. (The difference ·that I am invoking here· between •appearance and •experience is not the same as the difference between •illusion and •truth—·i.e. the difference between •how things seem and •how they are·. That's because •illusion ·or seeming· is nothing like •appearance: something's seeming to be the case always involves judgments about what is objectively the case; such judgments are always in danger of going wrong by taking the subjective to be objective, but in appearance there is no judgment of the understanding. This distinction is significant not only here but all through philosophy, because there is always confusion when what is said about 'appearances' is taken to be referring to illusion or seeming.)

Proposition 1

(a) 'That portion of matter is moving in a straight line in relation to that empirical space'—as distinct from 'The space is moving in the opposite direction in relation to the portion of matter'—is a merely possible predicate. **(b)** 'That portion of matter is moving in a straight line *period*', i.e. its movement is absolute, not thought of as a changing relation to matter outside it, is impossible. [Kant is assuming here that if you don't relate a moving thing to any body outside it you can't be relating it to an empirical space.]

Proof

In the case of a body x moving in a relative space y, these two:

(1) y is at rest and x is moving *this* way within it, and
(2) x is at rest and y is moving in *that* way—the opposite way—around it

tell the same story about what is objectively happening out there; they differ only in what they imply about the subject, ·the person whose experience is being reported on·. So there is no difference between them at the level of *experience*, only at the level of *appearance*. If the spectator puts himself into the space y, then he says that the body moves; if he puts himself (at least in thought) into another space z that encloses y, with x being at rest in relation to z, then he (the spectator) will say that space y is moving. Therefore, in experience. . . .there is no difference whatever between 556
(1) and **(2)**. [In a very repetitious passage, Kant belabours the point that **(1)** and **(2)** represent a pair of *choices* that one might make, not rival accounts of what is objectively the case. And yet they do apply conflicting predicates to x—'moving' and 'at rest'—from which Kant concludes:] Something that is in itself undetermined as regards two mutually opposed predicates is to that extent merely possible. So the straight-line motion of a portion of matter in empirical space—as against the opposite motion of the space—is in experience a merely possible predicate. This was **(a)** the first thing to be proved.

Next: For any •relation to be an object of experience, each of the related items must be an object of experience; this holds also for any •change of relation, including the special case of the relation-change that is •motion. Now pure space (in contrast to empirical space), i.e. absolute space (in contrast to relative space) is not an object of experience; basically it is nothing. So straight-line motion without reference to anything empirical, i.e. absolute motion, is utterly impossible. This was **(b)** the second thing to be proved. . . .

Remark

This proposition determines the modality of motion with respect to *phoronomy*—namely *possibility*.

Proposition 2

The circular motion of a portion of matter x, as against the opposite motion of the space, is an actual predicate of x.
557 On the other hand, the opposite motion of a relative space, taken as a substitute for the motion of x, is not an actual motion of x—at most it may *seem* to be an actual motion of that body, but this is a mere illusion.

Proof

Circular motion is (like every curved-line motion) a continuous change of •straight-line motion; and since •this motion is itself a continuous change of relation to external space; so circular motion is a *change of the change* of these external spatial relations, and is therefore a continuous arising of new motions. Now, according to the law of inertia, a motion can't start up without having an external cause. But the circulating body x at every point of this circle is (also by the law of inertia) endeavouring to proceed in the straight line at a tangent to the circle, and this straight-line motion acts *against* the external cause ·of x's circular movement·. [Re 'endeavouring', see the long note on page 19.] Hence every body in circular motion manifests by its motion a moving force. Now, the motion of the space, in contrast to the motion of the body, is merely phoronomic and has no moving force. Consequently, the judgment that here either the body is moved or else the space is moved in the opposite direction is a disjunctive one, by which, if the one member, namely, the motion of the body, is posited, then the other member, namely, the motion of the space, is excluded. Therefore, the circular motion of a body, in contradistinction to the motion of the space, is an actual motion. Even though according to the appearance [*Erscheinung*] •the motion of the space agrees with •the circular motion of the body, nevertheless in the complex of all appearances, i.e. of possible experience, the •former motion conflicts with the •latter; and hence the former is nothing but mere illusion [*Schein*].

Remark

This proposition determines the modality of motion with regard to dynamics—namely, *actuality*. For a motion that can't take place without the influence of a continuously acting external moving force exhibits—directly or indirectly—basic moving forces of matter, either of repulsion or of attraction. In connection with this topic, see Newton's scholium to the definitions at the start of his *Mathematical Principles of Natural Philosophy*. This makes it very clear that

the circular motion of rotating around a common centre, 558

and therefore also

the rotation of the earth on its axis,

can be known by experience even in empty space; which means that a motion that is a change of external relations in space can be empirically given, even though this space itself is not empirically given and is not an object of experience. This paradox deserves to be solved.

Proposition 3

In every motion of a body whereby it is moving with regard to another body, an opposite and equal motion of this other body is necessary.

Proof

According to the third law of mechanics (Proposition 4 on page 53), the communication of the motion of the bodies is possible only through the two-way causal interaction of their basic moving forces, and this two-way causal interaction is possible only through mutually opposite and equal motion.

So the motion of both bodies is actual. But the actuality of this motion doesn't come from. . . .the influence of external forces, but follows immediately and inevitably from a *concept*—the concept of *how something that moves relates to each other thing that can be moved by it*. So motion of the 'other thing' is necessary.

Remark

This Proposition determines the modality of motion with regard to mechanics, namely, *necessity*. It is immediately obvious that these three propositions determine the motion of matter with regard to its possibility, actuality, and necessity, and thus hence with regard to all three categories of modality. [This completes Kant's attempt to tie his four chapters severally to his four trios of categories.]

GENERAL REMARK ON PHENOMENOLOGY

So we have here three concepts that have to be employed in universal natural science, and which therefore have to be understood in precise detail—though the details are hard to pin down and hard to understand.

They are these:

(1) the concept of motion in relative (movable) space;

(2) the concept of motion in absolute (immovable) space;

559 **(3)** the concept of the across-the-board distinction between relative motion and absolute motion.

The concept of absolute space lies at the foundation of all of these. How do we come by this unusual concept, and why do we *have to* use it?

•The concept of absolute space can't be ·a concept of the **understanding**, because absolute space can't be· an object of **experience**—space without matter isn't an object of perception. But •it is a necessary concept of **reason**, and that gives it the status of an *idea*, but that is all it is—a mere idea. [See the note on 'idea' on page 9.] ·Here is how it comes into play·. For there to be even an *appearance* of motion, there has to be an empirical representation of the space with which the moving thing is changing its relation; but *that* space—the space that is perceived—must be *material* and therefore itself movable. [Kant says that this last 'therefore' depends on 'the concept of matter in general'. Perhaps he is referring to the equation of 'material space' with 'relative space' in the phoronomic Definition of 'matter' on page 7.] Now, we can't think of this space as moving except by thinking of it as contained in a more extensive space that is at rest. But this latter space can be related in just the same way to a still larger space . . . and so on to infinity, without ever arriving empirically at an immovable (immaterial) space with regard to which any portion of matter could be said to be outright moving or at rest. Rather, we have to keep changing our concept of these relational set-ups depending on what we are thinking of as moving relative to what. ·I'll say it again·:

> The condition for regarding something as at rest, or as moving, is always its being placed in a relative space—*always*, again and again ad infinitum, as we enlarge our view.

From this we can draw two conclusions: **(1)** All motion or rest must be merely relative; neither can be absolute. That is, matter can be thought of as moving or at rest only in relation to •matter and never in relation to •mere space without matter. It follows that absolute motion—·i.e. motion that doesn't consist in one portion of matter changing its relation to another portion·—is simply impossible. **(2)** For this very reason, there can't be, out of all the ever-wider concepts of motion or rest in relative space, one that is ·so wide as to be· valid for every appearance. ·To have such an all-purpose concept·, we have to make room in our minds for the thought of a space that isn't nested within any larger space, i.e. an

560 **absolute** space in which all relative motions are nested. In such a space everything empirical is movable,. . . .[5] but none can be valid as absolute motion or rest. . . . So absolute space is necessary not as a **concept** of an actual object but as an **idea** that is to regulate all our thoughts about relative motion. If we want all the appearances of motion and rest to be held together by a determinate empirical concept, we must put them within the framework of the idea of absolute space. [Actually, Kant writes that these appearances must all *auf den absoluten Raum reducirt werden* which literally = 'be reduced to absolute space'; but his meaning, in this sentence and the next, seems to be something about framing or handling-in-terms-of.]

Thus the straight-line motion of a body in relative space is handled in terms of [*reducirt auf*] absolute space **(i)** when I think of a body as being at rest and think of the relative space ·that it is in· as moving in the opposite direction—moving in non-empirical absolute space—·and **(ii)** when I think of the body as moving and the relative space as being motionless in absolute space·. The two ways of representing the situation are empirically exactly alike. By means of this representation all possible appearances of rectilinear motions which a body might simultaneously have are grounded in the concept of experience that unites them all, namely the concept of merely relative motion and rest.

According to Proposition 2 [page 61], circular motion can be experienced as actual motion, even if no external empirically given space comes into the story; so it seems to be absolute motion. ·I'll say that again, explaining it a little as I go·. A motion such as **(a)** the earth's rotation on its axis relative to the stars is an appearance that can be matched by **(b)** the opposite motion of the space of the stars, and these two are ·empirically· fully equivalent. But Proposition 2 forbids us ever to postulate **(b)** intead of **(a)**; so **(a)** is not to be 561
represented as externally relative—which sounds as though it is being assumed to be absolute.

But ·that's a mistake·. What we are dealing with here is the ·humdrum everyday· diference between what seems to be the case and what is really the case, not the ·metaphysical· distinction between relative space and absolute space. Employing the former distinction, we can and do have empirical evidence that the earth is really spinning and thus that the stars may be at rest, although the space they move in can't be perceived. The earth's circular motion doesn't present us with any appearance of change of place, i.e. any •phoronomic change in the earth's relation to the (empirical) space surrounding it; but it exhibits a continuous •dynamic change in the relations amongst portions of matter **within the space that it occupies**, and this change is provable by experience. For example, the attraction ·that holds the earth together· is constantly lessened by an endeavour to escape, ·i.e. by centrifugal force·; we know about this empirically, and it's a result of the earth's rotation, which shows that the rotation is real and not illusory. [Kant wrote this paragraph down to here as a single sentence.] Thus, for instance, we can represent the earth as spinning on its axis in infinite empty

[5] [Kant offers here a longish footnote, to the following effect: Any empirically knowable fact about something's moving can be construed either as a body moving in a relative space or a relative space moving around a body. In the context of phoronomy, these two are alternatives; we shouldn't think of them as a disjunction—'P or Q, one of which must be wrong'—because the difference between them is a difference of viewpoint, a difference in how the knowing *subject* relates to the state of affairs, not in what is *objectively* the case. In the context of dynamics, on the other hand, such a pair of propositions are rivals, which can't both be true. And in mechanics there is a different pattern again: When one body is rushing towards another, we must attribute an equal proportion of the total motion to each body. Kant is here presenting a trio of ways of understanding a certain thesis of the form 'P or Q': disjunctively (dynamics), distributively (mechanics), alternatively (phenomenology).]

space, and can produce empirical evidence for this motion even though it doesn't involve any phoronomic change (i.e. change in the appearance) in how the earth's parts relate •to one another or •to the space surrounding the earth. . . . ·Here is a description of one course of events that would provide such evidence·. I let a stone fall down a deep hole running to the centre of the earth; and I find that although gravity keeps taking it downwards, its fall continuously diverges from the vertical direction by tending towards the east; from which I conclude that the earth is rotated on its axis from evening to morning. . . . This is good enough evidence of the earth's actiually rotating in that way; and we don't get such evidence from •the change in the earth's relation to external space (the starry heavens). Why not? Because •that change is a mere appearance, which could come from either of two opposed causes—from the earth's spinning on its axis or from the stars revolving around the earth. . . . But the earth's rotation, even though it isn't a change of relation to empirical space (·I am now returning to the imagined case of a rotating world in a space that is otherwise empty·), isn't a case of absolute motion. Rather, it is a continuous change in how portions of matter •relate to to one another, so it really is only a case of •relative motion, although we represent it to ourselves as as happening in absolute space. And it's just because this movement of the earth is relative that it is true ·or actual· motion.

[Kant is here recalling us to his point that the line between
(a1) illusory and **(a2)** real or actual
is not the same as the line between
(b1) how things appear and **(b2)** how they are in themselves,
or the line between
(c1) relative space and **(c2)** absolute space.
He is emphasizing the difference by saying that the status of the earth's rotation as something **(a2)** real depends on its belonging to **(b1)** the realm of appearance.] ·Our evidence· that this rotation is true ·or actual· rests upon our encounter with the fact that parts of the earth outside its axis of rotation tend to fly off, i.e. the fact that any two parts of the earth that are exact antipodes of one another tend to move apart. . . . [Kant likens this 562
to a slightly different consideration that Newton used—two bodies joined by a cord and rotating, pulling on the cord—see his remark about a 'paradox' on page 61.]

As for the third proposition: to show the truth ·or actuality· of the motions of two bodies moving relatively to one another, showing this without reference to empirical space, we don't need to learn from experience about an active dynamical influence (of gravity or of a taut cord), though we needed this in the case of the second proposition. Rather, we can get this result from the mere dynamical *possibility* of such an influence, as a property of matter (repulsion or attraction). That possibility brings with it the result that any motion by one of the two bodies is matched by an equal and opposite motion of the other at the same time. indeed such action and reaction stem from mere concepts of a relative motion when this motion is regarded as in absolute space, i.e. according to truth. Therefore, this third proposition is, like everything adequately provable from mere concepts, a law of an absolutely necessary countermotion.

So there is no absolute motion even if a body in absolute space is thought of as moving in relation to another body. The motions of the two bodies are here not relative to the space surrounding them but only to the space between them, which is the sole determinant of their external relation to one another. . . . So these motions are only relative. Thus, absolute motion would have to be motion that a body has without a relation to any other matter, and the only candidate for this role would be the straight-line motion of the universe, i.e. of the system of all matter. ·It is easy to see why·: If outside of a portion of matter x there is any other matter, even if separated from x by empty space, then x's motion would certainly be relative. Thus, if you can show regarding any law of motion L that

> denying L implies that there is a straight-line motion of the whole universe,

that proves that L is absolutely necessary, because such
563 motion is utterly impossible. There is a law of that kind, namely the law of the reaction of portions of matter in all two-way causal interactions that depend on motion [see Proposition 4, page 53]. Any divergence from this law would ·consist in a shove in one direction without an equal shove in the opposite direction; so it would· create a straight-line movement of the common centre of gravity of all matter, and hence of the whole universe. No such result follows from the thesis that the entire universe rotates on its axis; so there is never any obstacle to thinking of the universe in this way, though I can't see any conceivable use for it.

Corresponding to the ·three· concepts of motion and moving forces, there are ·three· concepts of empty space. **(1)** What passes for 'empty space' (or 'absolute space') in the context of **phoronomy** really shouldn't be called empty space. It is only the *idea* [see the note on 'idea' on page 9] of a space from which I filter out all particular matter that would make it an object of experience, in order to think of it as the space within which every material or empirical space can move; this being something I want so as to think of every truth of the form 'x moves' not as predicating something of x alone but as relating x to something else. So this ·'ideal'· space belongs not to the •existence of things but merely to the •fixing of concepts; so no empty space ·on this pattern· exists. **(2)** in the context of **dynamics**, empty space is space that isn't filled, i.e. space in which things move without being resisted by other things, i.e. a space in which no repelling force acts. Such a space might be either •empty space within the world or •outside of the world (if the world is represented as limited). An empty space within the world can be further subdivided into

> **(a)** scattered all through the world, so that a part of the volume of any body may be empty space; and
> **(b)** occurring between bodies, e.g. as space between the stars.

This distinction is not theoretically deep, because it doesn't mark off different *kinds* of empty space but only different •places in the world where empty space might occur. Still, the distinction is put to use, because the two sides of it are used for different explanatory purposes. **(a)** Space within bodies is used to explain differences in the density of bodies; and **(b)** space between bodies is used to explain how motion is possible. It isn't necessary to **(a)** assume empty space for the first purpose, as I have shown in the General Remark on Dynamics [see pages 39–41 and 46–47]; but there's no way of showing that empty space is impossible because its concept is self-contradictory. Still, even if it can't be ruled out on merely •logical grounds, there might be general •physical grounds for banishing empty space from the doctrine of Nature. . . . Suppose that the following turned out to be the case (there are many reasons for thinking that it *is* the case):

> What holds bodies together is not •true but only •apparent attraction; what really holds a body to-
> gether is pressure from the outside, pressure from 564
> matter (the ether) that is distributed everywhere in the universe. What leads this matter to exert this pressure is *gravitation*, this being a basic attraction that all matter exerts.

If this is how things stand, then empty space within portions of matter would be impossible—not •logically but •dynamically impossible, and therefore physically impossible. Why? Because in this state of affairs every portion of matter would expand into the empty spaces assumed to be within it (because there's nothing here to resist such expansion), so that those spaces would always be kept filled up. As for **(b)** an empty space outside of the world (i.e. outside the totality of . . . .large heavenly bodies) would be impossible for the very same reasons. ·In the scenario we are exploring·, these large bodies are surrounded by ether which, driven by the attractive force, presses in on the stars and maintains them in their density. The further any portion of this ether is from the star-totality that we are calling 'the world', the less dense it is; this lessening of density continues ad infinitum as the distance grows; but it never gets to the point where the density is zero and that portion of space is therefore empty. [Kant does not try to explain why the density of portions of ether is proportional to their distance from 'the world'.] Don't be surprised that this elimination of empty space is in the meantime entirely hypothetical; the assertion that there *is* empty space doesn't fare any better! Those who venture to decide this controversial question dogmatically, whether for empty space or against it, basically rely on nothing but metaphysical suppositions, as you'll have noticed in the dynamics; and I had at least to show here that the question can't be answered by metaphysics. **(3)** Concerning empty space in a **mechanical** sense—i.e. the ·supposed· emptiness accumulated in the universe to provide the heavenly bodies with room to move—it is obvious that the possibility or impossibility of this doesn't rest on metaphysical grounds but on Nature's secrets (so hard to unravel!) concerning how matter sets limits to its own force of extension. . . .

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This brings us to the end of the metaphysical doctrine of body, and we end with *empty* and therefore with the *inconceivable*! On this topic, the doctrine of body meets the same fate as every other attempt by •reason to get back to the principles of the first causes of things. •It fails in these attempts because it brings to them its own nature, which is such that the only things •it can grasp are ones that are
specified as satisfying certain conditions, and yet it can never 565
be satisfied with anything conditioned. When it is gripped by a thirst for knowledge that invites it to reach for the absolute totality of all conditions, all it can do is to turn back from *objects* to *itself* in order to investigate and determine the ultimate boundary of its powers, instead of investigating and determining the ultimate boundary of things.

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·THE PASSAGE THAT CREATED A DEFEAT ON PAGE 15·

First case: Two motions in *one and the same line and direction* belong to one and the same point.

Two speeds AB and ab are to be represented as contained in one speed of motion. Let these speeds be assumed to be equal for the moment, so that AB = ab; then I say that they can't be represented at the same time in one and the same space (whether absolute or relative) in one and the same point. For, since the lines AB and ab designating the speeds are, properly speaking, the spaces they traverse in equal times, the composition of these spaces AB and ab = BC, and hence the line AC as the sum of the spaces, would have to express the sum of the two speeds. But neither the part AB nor the part BC represents the speed = ab, for they are not traversed in the same time as ab. Therefore, the doubled line AC, traversed in the same time as the line ab, does not represent the twofold speed of the latter, which, however, was required. Therefore, the composition of two speeds in one direction cannot be represented intuitively in the same space.

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·THE PASSAGE THAT CREATED A DEFEAT ON PAGE 51, FIRST IN ELLINGTON'S TRANSLATION:·

Now, the proper motion of matter is a predicate which determines such motion's subject (the movable) and with regard to matter as a multitude of movable parts indicates the plurality of the moved subjects (at equal velocity in the same direction); this is not the case with dynamical properties, whose quantity can also be the quantity of the action of a single subject (e.g., a particle of air can have more or less elasticity). Because of all of this it is clear that the quantity of substance in a matter must be estimated mechanically, i.e., by the quantity of the proper motion of the matter, and not dynamically, by the quantity of its original moving forces. Nevertheless, original attraction as the cause of universal gravitation can indeed provide a measure of the quantity of matter and its substance (as actually happens in the comparison of matters by weighing), although there seems to be laid at the foundation here not the proper motion of the attracting matter but a dynamical measure, namely, attractive force. But in the case of this force, the action of one matter occurs with all its parts directly on all parts of another matter; and hence the action is (at equal distances) obviously proportional to the number of the parts. Because of this fact the attracting body itself thereby also imparts the velocity of its proper motion (by means of the resistance of the attracted body). This velocity is directly proportional, in equivalent external circumstances, to the number of the attracting body's parts; because of this the estimation takes place here, as a matter of fact, mechanically, although only indirectly so.

·AND THEN IN FRIEDMAN'S:·

Now since the *inherent motion* of matter is a predicate that determines its subject (the movable), and indicates in a matter, as an aggregate of movables, a plurality of the subjects moved (at the same speed and in the same way), which is not the case for dynamical properties, whose magnitude can also be that of the action of a single subject (where an air particle, for example, can have more or less elasticity); it therefore becomes clear how the quantity of substance in a matter has to be estimated mechanically only, that is, by the quantity of its own inherent motion, and not dynamically, by that of the original moving forces. Nevertheless, *original attraction*, as the cause of universal gravitation, can still yield a measure of the quantity of matter, and of its substance (as actually happens in the comparison of matters by weighing), even though a dynamical measure—namely, attractive force—seems here to be the basis, rather than the attracting matter's own inherent motion. But since, in the case of this force, the action of a matter with all its parts is exerted immediately on all parts of another, and hence (at equal distances) is obviously proportional to the aggregate of the parts, the attracting body also thereby imparts to itself a speed of its own inherent motion (by the resistance of the attracted body), which, in like external circumstances, is exactly proportional to the aggregate of its parts; so the estimation here is still in fact mechanical, although only indirectly so.