ENTAILMENT

By Jonathan Bennett

I. INTRODUCTION

Following Moore, I use ‘P entails Q’ as a convenient shorthand for ‘Q can be deduced logically from P’, ‘From P, Q follows logically’, ‘There is a logically valid argument with P as sole premise and Q as conclusion’, and the like.[1] Apart from a minor point to be raised in Section XVI, distinctions within this cluster do not matter for present purposes.

An analysis of the concept of entailment is answerable to careful, educated uses of expressions such as those. An analysis which condemned nearly everything we say about what follows from what simply would not be an analysis of the common concept of entailment. If the concept were inconsistent, some common uses of it would be condemned; but only by standards established by the others.

C. I. Lewis maintained this: to say that P entails Q is to say that it is logically impossible that (P & ¬Q).[2] If Quine is right, then ‘entails’ and ‘impossible’ are as suspect as all other intensional terms. So perhaps they are; but their uses are not wholly without structure, and there are wrong ways of interrelating them. Lewis’s contention is about the internal geography of the intensional area, not its relations to the surrounding conceptual territory: it is an attempted analysis of one intensional expression in terms of another. I shall argue that Lewis was right, and also - by implication - that his thesis is helpful and clarifying - that is, that it is a genuine analysis.

As is well known, Lewis’s analysis implies that each impossible proposition entails every proposition. Accepting the analysis, I accept this result. For one thing, Lewis has an argument for it (I use ‘→’ to abbreviate ‘entails’):

(1) P & ¬P
(1) → (2) P
(1) → (3) ¬P
(2) → (4) P ∨ Q
(3), (4) → (5) Q.

If each step is valid and entailment is transitive, then each impossible proposition entails every proposition. Or, if some impossible propositions entail nothing of the form (P & ¬P), then we get the more modest result - which is still unacceptable to all but two of Lewis’s opponents[3] - that there are millions of impossible propositions which entail every proposition. For brevity, I shall refer to the thesis that each impossible proposition entails every proposition as ‘the paradox’.

There have been many attempts to show that the paradox is false, and many attempts to invalidate the above ‘Lewis argument’ (as I shall call it). The only writers I can discover to have come to the aid of the Lewis argument, or of the paradox, or of Lewis’s analysis of the concept of

----

[1] G. E. Moore, *Philosophical Studies* (London, 1922), p. 291.
[2] C. I. Lewis and C. H. Langford, *Symbolic Logic* (New York, 1932).
[3] Arnold F. Emch, ‘Implication and Deducibility’, *Journal of Symbolic Logic* I (1936); P. G. J. Vredenduin, ‘A System of Strict Implication’, ibid., IV (1939).

entailment, are Robert J. Richman, John Woods, and myself.[4] Many other philosophers think or suspect that Lewis is right, but they have not argued a case in print. Perhaps this is because they accept Lewis's analysis only reluctantly, seeing the paradox as something unpalatable which must be choked down because there is no convincing way of faulting the Lewis argument which supports it. I shall argue that this concedes too much to Lewis's opponents: the fact that his analysis generates the paradox is part of the case - the overwhelming case - *for* the analysis.

Wanting to marshal as complete a case as possible, rather than offer fragments to be stitched together with material already published, I shall repeat a number of points already made by Richman and Woods. The most helpful thing in the literature is a paper of Smiley's which explores, with a cogency and control which are rare in anti-Lewis writings, several alternatives to Lewis's analysis.[5] His paper, indeed, is not really part of the anti-Lewis literature:

> It may be that, like Moses, by tapping the rock too many times I have denied myself entry to the Promised Land; but the only 'Promised Land' that I can discern is the classical logic, paradoxes and all. (p. 234)

Of the four non-'classical' or non-Lewis accounts of entailment which Smiley presents, there is just one, dealt with in Section V below, which he rejects outright. Of two others (one mentioned in Section II below) he says:

> A possible use for these other calculi might be the development of non-mathematical formalised theories (deontic logic etc.), in which at present the paradoxical principles reappear in a quite intolerable way (ibid.).

This, while the word 'formalised' is there, is unobjectionable. My quarrel is with those who would wish, as Smiley does not, to claim that either of the calculi in question can be so interpreted as to give a satisfactory account of the common concept of entailment. Regarding the remaining one of his proposals, Smiley does make a certain limited philosophical claim which I shall dispute in Section VI below.

I have been helped by conversation with many philosophers, including J. E.J. Altham, Simon Blackburn, Sylvain Bromberger, Arthur W. Burks, Robert H. Ennis, Arthur N. Prior, J. F. Thomson and - above all - Timothy Smiley; and also by correspondence with Norman Kretzmann and the referee for this paper.

In Sections II-VII I shall defend the validity of the Lewis argument. In Sections VIII-XIV I shall try to meet the main objections to the paradox - the main reasons for saying undiagnostically that the Lewis argument must be wrong in some way. Then in Section XV I shall defend a second Lewis argument, and in Sections XVI-XVII an associated second 'paradox'. [Added in proof: Apologies to J. L. Pollock whose fine pro-Lewis 'Paradoxes of Strict Implication' is in *Logique et Analyse* 1966.]

Thus, I shall argue not directly for the truth of Lewis's analysis, but only for two of its most striking consequences - namely, the 'paradoxes'. But the latter have in fact been the main obstacles to acceptance of the analysis; and so my arguments, if successful in their immediate purpose, will also count fairly heavily in favor of Lewis's analysis of the common concept of entailment.

----

[4] Robert J. Richman, 'Self-contradiction and Entailment', *Analysis*, XXI (1960-1961). John Woods, 'Relevance', *Logique et Analyse* VII (1964); 'On how not to Invalidate the Disjunctive Syllogism', ibid., VIII (1965); 'On Arguing about Entailment', *Dialogue*, III (1965). Jonathan Bennett, 'Meaning and Implication', *Mind* , LXIII (1954).

[5] T. J. Smiley, 'Entailment and Deducibility', *Aristotelian Society Proceedings*, LIX (1958-1959).

II. FROM CONJUNCTIONS TO CONJUNCTS

The first move of the Lewis argument, from (1) to (2), has been challenged on the grounds that (P & R) → P is not unrestrictedly true. (The analogous objection to the move from [1] to [3] need not be separately discussed.) Nelson, indeed, has taken the strong line that every instance of that principle is false.[6]

To assess this claim, construed as one about the common concept of entailment, we must know how to apply it to what is said in plain English. When does an English sentence express something of the form (P & R)? There seem to be only two possible answers:

(i) A proposition has the form (P & R) if it *could* be expressed by two sentences linked by an 'and'. But by that criterion Nelson's denial rules out all the entailments we ordinarily accept, does not count any of our ordinary arguments as valid; for what makes an argument valid in any plain case is just the fact that the premise expresses what the conclusion does plus (perhaps) a bit more. In such cases as the move from *x is red* to *x is colored* there is no snappy way of expressing the 'bit more'; but there is always a clumsy way of doing so - for example, *If x is red then x is colored* or *x is not both colored and not red*. I am here depending upon the general principle that if S → P then S is logically equivalent to (P and ¬(P & ¬S). If Nelson accepts this and denies that (P and ¬(P & →S)) → P, he must deny that S → P. That is, he cannot allow that any entailments hold.

Nelson would probably reject the principle that if S → P then S is equivalent to (P and ¬(P & ¬S)), denying, for example, that *x is red* is equivalent to *x is colored and x is not both colored and not red*. That would block the argument of the preceding paragraph. But it would also, unless accompanied by a detailed positive account of logical equivalence such as Nelson does not give, block any attempt to assess his position when interpreted according to criterion (i). The criterion itself rests heavily on the notion of equivalence: the question of whether S 'could be expressed' by two sentences linked by 'and' involves questions of the form 'Is S equivalent to the proposition that . . .?' with the blank filled by a suitable 'and'-using sentence. To be able to do anything with the criterion, then, we must either be allowed to use the notion of logical equivalence with normal liberality as I have done, or else be told precisely how that liberality is to be restricted. So, although my argument in the preceding paragraph is not decisive, I submit that if Nelson opts for criterion (i) the ball is on his side of the net.

(ii) A bit of English expresses a proposition of the form (P & R) only if it does consist of two sentences linked by 'and'. By this criterion, Nelson's denial implies that *x is rectangular* is entailed by *x is square* but not by *x is rectangular and x is equilateral*: argument-validity now depends not just upon what the premises say but upon how they say it. Now, we do indeed sometimes praise or criticize an argument for reasons which concern the way it is worded; but it is quite another matter to use such assessments as those as the basis for our whole theory of validity. If the latter is to be given such a basis, we must put away our Ps and Qs (which we use precisely in order to bypass details of wording), and must embark on a kind of philosophical inquiry which, so far as I know, no one has even begun to attempt. Criterion (ii), in short, adopts a revolutionary approach to argument-validity but is not accompanied by any of the spadework needed to show that the revolution could possibly succeed.

----

[6] E. J. Nelson, 'Intensional Relations', *Mind*, XXXIX (1930).

Why should anyone think that P does not follow from (P & R) if the latter is so worded that an 'and' neatly splits off P from R? My guess - and one is forced to guess - is that an argument-move which is thus worded looks too trivial to count as a move, has an air of 'not getting anywhere'. There could hardly be a worse motivation for the view in question; for a prime requirement of a cogent, fully expressed, deductively valid argument is that each separate step shall be so trivially or obviously valid that it looks less like a 'move' than like a stammer.

Suppose that these objections could be met. Suppose indeed that Nelson's position, applied according to criterion (ii), were actually correct. Indefinitely many Lewis-type paradoxes would still be left standing. While unable to maintain that 'Grass is green and it is not the case that grass is green' entails everything, we should have no reason for denying that 'My scarf is a triangular square' entails everything. But the usual reasons for rejecting the Lewis analysis, inadequate as they are, have as much force against the latter as against the former.

By saying that every instance of $(P \& R) \rightarrow P$ is false, Nelson landed himself in difficulties which could be seen without asking how '&' relates to 'and'. For example, he had to restrict the innocent principle $(((P \supset Q) \& (Q \supset R)) \rightarrow (P \supset R)$, because when P is substituted for Q and for R in this the result has the form $(P \& P) \rightarrow P$ which is of the form $(P \& R) \rightarrow P$. Smiley points out that such troubles could be avoided by weakening Nelson's position: instead of saying that every instance of $(P \& R) \rightarrow P$ is false, simply decline to allow that every instance of it is true.[7]

This certainly improves on Nelson's position, considered as a possible stand in the development of a formal logic of entailment. Considered as a suggestion about the common concept of entailment, however, it says too little to be assessed. Someone who seeks to make philosophical capital out of Smiley's suggestion must tell us what English sentences he takes to express propositions of the form (P & R), give examples of sentences of that kind for which he regards the corresponding move from (P & R) to P as invalid, and defend what he says about those examples. Then we shall have something to argue with.

Analogous remarks apply to Smiley's weakening of the over-strong position of Nelson's discussed in my next section.

A very different line of attack on the move from (1) to (2) will be discussed in Section V below.

III. FROM DISJUNCTS TO DISJUNCTIONS

The denial that $(2) \rightarrow (4)$, based on the rejection of $P \rightarrow (P \vee Q)$, stands or falls with the rejection of $(P \& R) \rightarrow P$; for the two are inter-derivable by means of logical principles - such as contraposition - which so far as I know have never been challenged by opponents of the paradox.

This link between the two rejections is mirrored by a similarity in the arguments against them. When does something in English have the form $(P \vee Q)$? Again, there seem to be only two prima facie possible answers. (i) A proposition is of that form if it ***could be*** expressed by two sentences linked by an 'or'. But Nelson's rejection of every instance of $P \rightarrow (P \vee Q)$, on that criterion for its application, rejects every entailment. For suppose that $P \rightarrow S$; then S is logically equivalent to (P or $(\neg P \& S)$); the view under discussion, in its criterion (i) version, denies that $P \rightarrow$ (P or $(\neg P \& S)$); and so it must deny that $P \rightarrow S$ and hence reject every entailment. To this argument, as to the analogous one in Section II, I have to add that Nelson can escape it; for he can reject the principle

----

[7] Smiley, op. cit., p. 249.

that if P $\rightarrow$ S then S is equivalent to (P or ($\neg$P & S)). If he does, then I need to know more before I can say more. (ii) Alternatively, a sentence expresses something of the form (P $\vee$ Q) only if it ***does*** consist of two sentences linked by an 'or'. This, unattractively, makes validity dependent upon details of how arguments are worded. Also, the rejection of P $\rightarrow$ (P $\vee$ Q), applied by this criterion, seems to have no motivation other than that the argument-moves it condemns are all trivially or elementarily valid - which is a poor reason for condemning them as invalid.

In any case, we can construct indefinitely many Lewis-type arguments in which no move counts, by criterion (ii), as having the form P-to-(P $\vee$ Q). I remarked in Section II that the criterion (ii) rejection of (P & R) $\rightarrow$ P is not called into play by arguments of the form:

(a) My scarf is a triangular square
(a) $\rightarrow$ (b) My scarf is triangular
(a) $\rightarrow$ (c) My scarf is square
(c) $\rightarrow$ (d) My scarf is square or Q
(b) $\rightarrow$ (e) My scarf is not square
(d), (e) $\rightarrow$ (f) Q,

where Q can be any proposition you like. The criterion (ii) rejection of P $\rightarrow$ (P $\vee$ Q) now condemns the move from (c) to (d); but we can get around that, too - for example, by replacing (d) by 'My scarf is rectangular' and (f) by 'My scarf is oblong'. I doubt if Lewis's opponents would be content to allow that (a) entails 'My scarf is oblong'. It may be said that the revised argument lacks the mad implausibility of the abstract Lewis argument because in the former both premise and conclusion have the same topic - namely, the shape of my scarf. That raises issues which I shall discuss in Section XIII below.

IV. THE FINAL MOVE

The move from (3) and (4) to (5) in the Lewis argument remained unchallenged in the literature until fairly recently, when Anderson and Belnap questioned it. More precisely, they have maintained that the Lewis argument is fallacious through ambiguity: P $\rightarrow$ (P $\vee$ Q) holds only where '$\vee$' has a truth-functional sense, while ((P $\vee$ Q) & $\neg$P) $\rightarrow$ Q holds only where '$\vee$' has a stronger, 'intensional' sense; and so there is no one kind of disjunction in terms of which the whole Lewis argument goes through.[8] In discussing this position, I shall use '$\vee$' truth-functionally, adopting 'V' to symbolize the supposed intensional disjunction.

Anderson and Belnap claim support from facts about the uses of 'or' in English. There is a sense of 'or' - corresponding to 'V' - in which 'P or Q' entails that P and Q are so related that one is entitled to say 'If P had not been true, Q would have been true' or 'If Q had not been true, P would have been true' or the like. I shall use ($\neg$P $\Rightarrow$ Q) to symbolize the statement that P and Q are so related as to justify subjunctive (sometimes counterfactual) conditionals such as these. So, (P V Q) entails ($\neg$P $\Rightarrow$ Q). But clearly P does not entail ($\neg$P $\Rightarrow$ Q): that the sky is blue does not entail that if it were not it would be yellow. It follows, then, that P does not entail (P V Q).

Some genuine linguistic facts do lie behind all this. I should not care to embody them in the claim that 'or' is ambiguous, but let that pass in the meantime. Let us grant that (4) in the Lewis argument is ambiguous, and that if it means (P V Q) then it is not entailed by (2). To complete their case, Anderson and Belnap have also to maintain that if (4) is read truth-functionally, so that

----

[8] Alan Ross Anderson and Nuel D. Belnap, Jr., 'Tautological Entailments', *Philosophical Studies*, XIII (1962).

$(2) \rightarrow (4)$, then it is false that $((3) \& (4)) \rightarrow (5)$. This is prima facie extremely implausible, yet I cannot find that they offer any arguments for it. Of course I am going along with the denial of

$$(P \vee Q) \rightarrow (\neg P \Rightarrow Q);$$

but what is now in question is the denial of

$$((P \vee Q) \& \neg P) \rightarrow Q,$$

which is quite different and, one would have thought, not to be rejected except upon the basis of strenuous arguments. The best guess I can make is that Anderson and Belnap have been attracted by the following line of thought.

They explain the alleged intensional sense of 'or' as one which requires 'relevance' between the disjuncts, and the general idea seems to be this: someone who says 'P or Q' expresses (P V Q) only if he has grounds for accepting $(P \vee Q)$ over and above any grounds he may have for accepting P or for accepting Q or for accepting (P & Q). If this is a sufficient as well as a necessary condition for a disjunction's being intensional - that is, for its using 'or' to mean 'V' - then 'or' will mean '$\vee$' only when the condition in question is not satisfied. That is, someone who says 'P or Q' and thereby expresses a truth-functional disjunction will be someone who accepts 'P or Q' only because he accepts P or accepts Q or accepts (P & Q). That suggests the following argument against the principle that $((P \vee Q) \& \neg P) \rightarrow Q$. Someone who accepts the premise $(P \vee Q)$ - that is, the premise 'P or Q' construed truth-functionally - either (a) accepts it because he already accepts P, or (b) accepts it because he already accepts Q. Suppose now that he employs the disputed principle: to his premise $(P \vee Q)$ he adds the further premise $\neg P$, and thence infers Q. If (a) is the case, he is caught in a contradiction; if (b) is the case, then his inferential procedure is just a useless ramble from Q to Q. In short, every use of the principle $((P \vee Q) \& \neg P) \rightarrow Q$ involves either a logical mistake or a waste of time; and the principle ought therefore to be rejected.

But if that is a sufficient reason for rejecting the principle, then the principle $P \rightarrow P$ ought also to be rejected; yet Anderson and Belnap accept it.

There are other difficulties as well. Suppose that you, knowing Smith to be a U.S. senator, tell me 'Smith is a U.S. senator or Smith is a U.S. representative'; and I, finding that Smith is not a representative, conclude that he is a senator. Anderson and Belnap must either say that I have argued invalidly, or say that although you meant $(P \vee Q)$ by your disjunction I *had* to understand (P V Q) by it. They must in fact be prepared to countenance the latter alternative, and not just because the former is clearly wrong: since the distinction between $\vee$ and V has to do with the grounds one has for accepting the given disjunction, there are bound to be cases where a speaker and hearer cannot attach the same sense to a disjunctive form of words. This fact is one pointer to what is wrong with embodying these linguistic data in the claim that 'or' is ambiguous. It is masked by saying that in an intensional disjunction 'there is relevance' between the disjuncts; for this suggests a person-neutral demarcation, as though the distinction between $(P \vee Q)$ and (P V Q) had to do only with what P and Q are. (This suggestion is encouraged by the way 'relevance' is formalized by Anderson and Belnap in one of their systems.[9]) But if the criterion for plain-English 'intensional disjunctions' is not person-relative in the way I have indicated, then I have not even an approximate idea of what an 'intensional disjunction' is supposed to be. If it is person-relative, on the other hand, then this is an excellent reason for denying that what we have here are two senses of 'or'.

----

[9] Alan Ross Anderson and Nuel D. Belnap, jr., 'The Pure Calculus of Entailment', *Journal of Symbolic Logic*, XXVII (1962).

Also, it should be noted that sentences of the form ‘P or Q’ seldom occur, in arguments of the disputed form ***or in any other way***, except in circumstances which would lead Anderson and Belnap to say that what is expressed is an intensional disjunction. If I am in a position to say P, it is not likely to be sensible for me to utter instead the longer and weaker ‘P or Q’. Issues raised by that consideration will be discussed in Section XIII below, but I wonder what Anderson and Belnap would make of it.

I also wonder whether their thesis is meant to apply only to disjunctions using ‘or’ or the like. Does ‘Smith is a member of the U.S. Congress’ express a disjunction? If so, then is it an ambiguous one whose exact meaning depends upon whether the speaker knows that Smith is a U.S. senator? If it does not express a disjunction at all, and so does not fall within the scope of the Anderson-Belnap thesis, then does the latter allow the multitude of Lewis-type results which can be achieved with P-to-($P \vee Q$) moves, or at any rate ***weakening*** moves, which are not so worded as to involve ‘or’?

(I once devoted an entire review of a mainly technical paper by Anderson and Belnap to assaulting some of their passing philosophical remarks about entailment.[10] Lest I reinforce the false impression which that might have given, let me emphasize that, unlike most of Lewis’s opponents, Anderson and Belnap have offered highly developed formal embodiments of their views about entailment.)

V. ‘CONTRADICTIONS ENTAIL NOTHING’

Lewis’s analysis implies that if P is impossible then we cannot sort propositions out into those which P entails and those which it does not. This much has been agreed to by Strawson and Körner, unlike the majority of Lewis’s opponents who maintain that P entails some propositions but not others.[11] But where Lewis takes this position because P entails everything, Strawson and Körner have taken it on the grounds that P entails nothing. I shall argue that if their position is to escape a certain overwhelming objection, it must become one which differs only slightly, and for the worse, from Lewis’s.

The crucial point is that we do sometimes work with a set of premises S whose modal value is initially unknown, and learn that S is impossible precisely by finding that it entails ($P \,\&\, \neg P$) or something else which is logically impossible. (There is no difference that matters for present purposes between deriving something from the set of premises R_1, R_2, . . ., Rn, and deriving it from the single premise $R_1 \,\&\, R_2 \,\&\, . . . R_n$.) To condemn all such procedures outright would be not only to deny our right to infer from impossible propositions but also to deprive us of an indispensable technique for discovering, in hard cases, that given propositions ***are*** impossible (and for discovering, in hard cases, that given propositions are necessary). If Strawson and Körner are to avoid such iconoclastic puritanism, they will have to say something like the following. Without knowing P’s modal value we can know that P either entails or (let us say) quasi-entails Q, for the relation of entailing-or-quasi-entailing is governed by principles which do not restrict the modal values of the related propositions. For example, all conjunctions entail-or-quasi-entail their

----

[10] Review of ‘The Pure Calculus of Entailment’, *Journal of Symbolic Logic*, XXX (1965), 240-241.

[11] P. F. Strawson, ‘Necessary Propositions and Entailment-statements’, *Mind*, LVII (1948), 186; S. Körner, ‘On Entailment’, *Aristotelian Society Proceedings*, XLVII (1946-1947), 158. See also John Watling, ‘Entailment’, *Aristotelian Society Proceedings*, supp. vol. XXXII (1958), 148-150.

conjuncts, though only consistent ones entail their conjuncts. Another and more pertinent example: if S entails-or-quasi-entails Q, and Q is impossible, then S is impossible. Given the latter principle, we can use a ***reductio ad absurdum*** technique to discover that S is impossible, without having to say that S actually entails anything; for the ***reductio*** goes through just so long as S entails-or-quasi-entails something impossible.

Only in some such form as that has the Strawson-Körner view any claims to acceptance. But then it amounts to nothing but three terminological recommendations: instead of 'P entails Q' say 'P entails or quasi-entails Q'; instead of 'P entails Q and is consistent' say 'P entails Q'; and instead of 'P entails Q and is inconsistent' say 'P quasi-entails Q'. These seem to be bad advice, and certainly they are uninteresting advice.

The relevant papers by Strawson and Körner are not very recent, and their authors might now disown the view I have been attacking; but there is a special reason for launching the attack. Confronted with the nakedly schematic form of the Lewis argument, many are inclined to protest that the argument is radically flawed because it gives with one hand what it takes back with the other, or because its premise affirms something whose denial is essential to the whole concept of argument-validity. Such remarks, cleansed of metaphor and rhetoric, embody this truth: the Lewis argument is radically flawed because it has a logically impossible premise. But that much is common ground, and does not in itself count against Lewis's view that the argument is valid. To think that it does is to make the assumption - which I have therefore wanted to controvert - that impossible propositions do not entail anything.

(Von Wright has sought to use the phenomenon of ***reductio ad absurdum*** arguments against Lewis, maintaining that such arguments require a distinction between what is and what is not entailed by a given impossible proposition.[12] As Smiley points out, this is incorrect.[13] To show by ***reductio*** that P is impossible, we have only to show that $P \rightarrow \neg P$, or that $P \rightarrow Q$ where Q is known to be impossible. If P entails everything else as well, ***tant mieux***.)

VI. TRANSITIVITY AND AMBIGUITY

The last resort is to deny that entailment is transitive. One who takes this line might be expected to adduce evidence that careful, intelligent, literate people do generally decline to allow that, just because R follows from Q which follows from P, therefore R follows from P; but no one has even tried to produce evidence for this extraordinary view. The fact that some philosophers say that entailment is not transitive is not evidence. If a good enough philosopher says this, we may be led to think that, implausible as the view is, some case can be made for it; but then We shall wait to hear the case.

Some have said, more modestly, that 'entails' has one sense in which entailment is not transitive. If this thesis is to touch Lewis's position, several conditions must be met. (i) The allegedly non-transitive sense of 'entails' must be explained; and sometimes not even this much is done. (ii) The Lewis results must be shown not to hold for the non-transitive sense of 'entails'. An attempt by Von Wright and Geach failed to satisfy this minimal condition.[14] (iii) The non-

----

[12] G. H. Von Wright, *Logical Studies* (London, 1957), p. 174.
[13] Smiley, op. cit., pp. 237-238.
[14] G. H. von Wright, op. cit.; P. T. Geach, 'Entailment', *Aristotelian Society Proceedings*, supp. vol. XXXII (1958), 64. See P. F. Strawson's review of von Wright, *Philosophical Quarterly*, VIII (1958), 375; and Jonathan Bennett, 'A Note on Entailment', *Mind*, LXVIII (1959).

transitive sense must be one which 'entails' does have - that is, which is possessed by each expression in the cluster for which 'entails' is a shorthand.

Smiley has defined a relation (I shall call it S-entailment) which satisfies the first two conditions.[15] He defines it in a formal language, but it can be generalized to cover informal contexts as well. S-entailment is, in effect, a formalized version of one-step entailment. Let us call a principle of inference 'basic' if one application of it can lead from a contingent premise to a contingent conclusion: then 'P S-entails Q' can be defined as 'Q can be derived from P by a single application of a basic principle of inference'. This validates every step in the Lewis argument, but does not validate the move from (P & ¬P) to Q,.

So far, so good; but what about condition (iii)? Smiley rightly says that S-entailment is not merely a factitious construct: it does play a role in our logical thinking because ordinary transitive entailment is the ancestral of S-entailment, and this entitles Lewis's critics to an interest in the latter. (If ordinary entailment were not transitive, incidentally, it could not be the ancestral of anything.) But that is a far cry from maintaining that S-entailment expresses a sense which 'entails' and so forth do sometimes have, or, in Smiley's phrase, that S-entailment is 'a satisfactory reconstruction of an intuitive idea of entailment'. It does perhaps reconstruct the idea of intuitive entailment - that is, of obvious or elementary or one-step entailment; but that is not what has to be shown.

Furthermore, S-entailment could bring solace to Lewis's critics only if it were shown that at least one step in the Lewis argument is valid *only* if 'valid' is understood in terms of S-entailment. Smiley does not try to show this, because he rightly does not believe it.

VII. AMBIGUITY AND INCONSISTENCY

The thesis that 'entails' is ambiguous in some way or other (never mind the details) might be defended on the grounds that many people are inclined to accept each step in the Lewis argument, and to agree that entailment is transitive, and yet to deny the paradox. If their position is to be consistent, it may be argued, they must be using 'entails' ambiguously - in one sense when they say some of these things, and in another when they say the rest.

But a charge of inconsistency is not adequately met by a plea of ambiguity unless there are independent grounds for the plea. If I say 'Smith earns his living doing legal work', and later say 'Smith is an old scoundrel - he hasn't done a legal thing for years', I can defend myself against a charge of contradiction by pleading that 'legal' has two senses. And I can support this by saying what the two senses are '(law-abiding' and 'pertaining to the law') and indicating how to tell which sense is involved in a given context. Without this extra backing, my plea that 'legal' is ambiguous would be mere word-spinning: it would amount to saying that I have not contradicted myself, because 'legal' has two senses, as is shown by the fact that if it has not then I have contradicted myself. In the case of the alleged ambiguity of 'entails' the extra backing seems to be unavailable.

This suggests a still more modest view - namely, that our concept of entailment is inconsistent. Someone might maintain that our careful and considered uses of 'entails' cannot all be salvaged with 'entails' read univocally, and agree that he cannot go on to defend a plea of ambiguity; thence concluding that the common concept of entailment is inconsistent. If he were

----

[15] Smiley, op. cit., pp. 238-242.

right, we should have to select the most satisfactory subset from all the entailment principles we are initially inclined to accept, and relinquish the rest. But if our concept of entailment is inconsistent, it is so because (i) our natural inclination to deny Lewis-type results clashes with (ii) everything else we are inclined to say using 'entails' and the rest; in which case the rational course is to 'select' (ii) and relinquish (i).

Is the common concept of entailment inconsistent? Those who accept each step in the Lewis argument, and agree that entailment is transitive, yet deny the paradox, are indeed guilty of inconsistency; but that is a fact about them rather than about entailment. The fact that many people, while accepting each step in the Lewis argument, and so forth, are strongly inclined to deny the paradox, may look more like evidence for inconsistency in the concept; though even here less charitable diagnoses are possible. But if it is clear that we ought to accept the paradox, it does not matter much whether this acceptance is described as our rectifying a previously inconsistent concept or as our handling more competently the consistent concept we have had all along.[16]

Attempts to invalidate the Lewis argument, then, seem to be doomed. Some have accepted this conclusion only with reluctance, feeling that there ought to be something wrong with the argument. Let us consider why.

VIII. COUNTERINTUITIVENESS

It is sometimes said that $(P \& \neg P) \rightarrow Q$ is 'counterintuitive'or 'unacceptable', 'totally implausible', 'outrageous' or the like. To call the paradox 'counterintuitive' is, apparently, to say that it seems to be logically false. Perhaps it does, but then so does 'There are as many odd prime numbers as odd numbers', yet this is true by the only viable criterion of equal-numberedness we have. Our resistance to it can be explained: most of our thinking about numbers involves only finite classes, and it never is the case that there are as many Fs as Gs if the Fs are a proper subclass of the *finite* class of Gs. If someone said, 'Yes, I see all that, and I have no alternative criterion of equal-numberedness; but I still don't accept that there are as many odd prime numbers as odd numbers', we should dismiss this as mere autobiography.

Yet consider what happens when entailment is in question. Lewy, for example, has produced an apparently valid proof from true premises of something I shall call P: '*There are exactly ten brothers* and *there are exactly ten brothers* entails *There are exactly as many brothers as male siblings*'. This, as Lewy notes, is a consequence of Lewis's analysis (see Section XV below), though his own route to it owes nothing to Lewis. My concern is not with P itself but with the manner of Lewy's rejection of it. This is typical of appeals to 'intuition' and the like by Lewis's opponents, except for the fact (which is my somewhat unfair reason for selecting it) that it is more explicit and candid than the average:

> Have I merely shown that [P is true] though at first sight it does not seem to [be] so? I do not really think so. Of course, it often happens that a proposition does not seem to be entailed by another proposition, . . . but then a proof is produced that it is so entailed. And a man who denied that the entailment in question held, would be said not to have understood the proof. But in the [present] case the position does not seem to me to be at all like this . . . I think I can understand quite well the relevant 'proof', and yet I am not

----

[16] For a fuller treatment of the topic of Section. VII, see Woods, 'On Arguing about Entailment', pp. 416-421.

> really willing, even after considerable reflexion, to accept the alleged conclusion. In other words, to put the point briefly, [P] is highly counter-intuitive, not just surprising.[17]

Lewy rests everything on the distinction between surprisingness and counterintuitiveness, yet he gives no content to ‘P is counterintuitive’ except the autobiographical report ‘I am not really willing, even after considerable reflexion, to accept P’. No arguments are offered - not even a report on the structure and content of the ‘considerable reflexion’.

Counterintuitiveness can operate as a final court of appeal, down at the ground-floor level where ‘It is counterintuitive’ means ‘Everyone would agree that it is a patent abuse of the language’. But what can it amount to at the level of controversial general principles in logic or philosophy? At most, a judgment of counterintuitiveness may lead to a search for arguments against the thesis in question - for instance, the thesis that $(P \& \neg P) \rightarrow Q$. This particular search has been under way for decades, and has so far most miserably failed. Yet still we are told that the paradox ought to be rejected because it is counterintuitive; as though the hunch which motivated the search for arguments could, merely by surviving the failure to find any, count as an argument itself.

IX. MEANING-CONNECTION

It is often said that the paradox infringes the principle that if $P \rightarrow Q$ then P must be ‘connected in meaning’ with Q.[18] This complaint has never yet been accompanied by an elucidation of ‘meaning-connection’ (as distinct from a suggested representation of it in an extremely limited formal language), let alone by an attempt to show that in the given sense of ‘meaning-connection’ it is true both that (a) where there is an entailment there is a meaning-connection, and that (b) for some Q. there is no meaning-connection between $(P \& \neg P)$ and Q.

The following thesis is arguable. To establish a really elementary entailment is to note certain facts about meanings: to know that x is a triangle entails x has three sides is to know certain facts about the meanings of ‘is a triangle’ and ‘has three sides’ or equivalent expressions in some other language. More specifically, it is to know that saying of something that it ‘is a triangle’ is just one way of saying, perhaps among other things, what is also expressed by saying that it ‘has three sides’. This line of thought roughly locates a notion of meaning-connection (which we might call tight meaning-connection) and gives it a central relevance to entailment as a whole: entailment is the ancestral of elementary entailment, and elementary entailments hold only where there are tight meaning-connections. Lewis’s opponents, when they express themselves on the epistemology or metaphysics of logic, tend to reject this linguistic or ‘conventionalist’ view; yet the latter may nevertheless lurk behind the confident assumption that there is a sense of ‘meaning-connection’ in which (a) there is a meaning-connection wherever there is an entailment, and (b) for some Q there is no meaning-connection between $(P \& \neg P)$ and Q. But it will not do, for (a) is certainly false on this account of meaning-connection. Even if it is true that if $P \rightarrow Q$. then there are some $R_1, \ldots, R_n$ such that P is tightly meaning-connected to R_1, and R_1 to R_2, . . ., and R_n to Q, it simply does not follow that if $P \rightarrow Q$, then P is tightly meaning-connected to Q.

----

[17] C. Lewy, ‘Entailment and Propositional Identity’, *Aristotelian Society Proceedings*, LXIV (1963-1964), 111.

[18] Nelson, op. cit.; Austin Duncan-Jones, ‘Is Strict Implication the same as Entailment’? *Analysis*, II (1935); Charles A. Baylis, ‘Implication and Subsumption’, *Monist*, XLI (1931). For other references, see Bennett, ‘Meaning and Implication’.

‘But if there is a tight meaning-connection between P and R_1, between R_1 and R_2, . . ., and between R_n and Q, then there must be ***some*** meaning-connection between P and Q.’ To assess this conditional, we must understand the phrase ‘some meaning-connection’ which occurs in its consequent. If it requires a separate elucidation, then we are back to square one. Alternatively, it may be suggested that ‘tight meaning-connection’ obviously generates a sense for ‘(not necessarily tight) meaning-connection’, and that this latter sense, even if we are at a loss to explain it, is obviously one which makes the disputed conditional true. This invites us to steer by our ‘intuitions’, and I for one must decline. I suggest, though, that the state of mind in which the disputed conditional seemed obviously true would be one in which the following would also seem obviously true: ‘If x_1 closely resembles x_2, . . ., and x_{n-1} closely resembles x_n, then x_1 somewhat resembles x_n’· Yet the latter is false unless ‘resembles’ is so used that everything somewhat resembles everything else.

The disputed conditional - that if there is a tight meaning-connection between each proposition and the next, then there is some meaning-connection between the first and the last - might be made true by definition of ‘meaning-connection’. This need not be a lunatic procedure: it might be convenient so to use ‘meaning-connected’ that ‘P is meaning-connected with Q’ is just one way of saying that there are some R_1, . . ., R_n such that there is a tight meaning-connection between P and R_1, . . . , and between R_n and Q. But this secures (a) that wherever there is an entailment there is a meaning-connection, at the price of losing hold of the other needed premise - namely, (b) that for some Q there is no meaning-connection between (P & ¬P) and Q. For, on this account of ‘is meaning-connected with’, the alleged lack of meaning-connection between (P & ¬P) and Q can no longer be claimed to be obvious at a glance: the question of whether there is a meaning-connection is now the question of whether (P & ¬P) → Q.

Perhaps those who think it useful to demand a ‘meaning-connection’ between entailment-related propositions think of entailment as a kind of push, and see the Lewis results as postulating action at a distance. I can find no basis for the idea that entailment is a kind of operative tie: certainly, to say that P ¬ Q is not to say that P’s truth would bring out Q’s truth, or that the assertion of P would necessitate, or even obligate one to, the assertion of Q.

The demand for a ‘meaning-connection’ between premise and conclusion may refer not to the cloudy notion I have been pursuing in this section, but rather to the entirely different notion of ‘topic-linkage’ which I shall discuss in Section XIII.

X. OTHER ARGUMENTS

One strategy for arguing against the paradox is to defend a principle of the form ‘If A → B, then φ(A,B)’ and then to argue for the falsity of φ((P & ¬P),Q) for some Q. I have confuted one version of this strategy, in which φ(A,B)is ‘A is connected in meaning with B’; but other values of φ might serve better.

I have heard Lewis’s analysis being scoffed at on the grounds that such statements as ‘If triangles were four-sided then bananas would be purple’ are patent rubbish. This criticism seems to presuppose the general thesis that if A → B then the corresponding subjunctive conditional will not be rubbishy or peculiar. A very modest testing of this principle, however, shows that it is false. The fact is that any conditional which begins ‘If triangles were four-sided . . .’ will strike one as implausible, weird, unsatisfactory: the principle in question refutes Lewis only if it shows that no obviously impossible proposition entails anything.

The thesis that obviously impossible propositions entail nothing has its attractions. In particular, it does not conflict with any serious use of ***reductio ad absurdum*** - that is, any use of it in an argument designed to expose an ***un***obvious impossibility. But if we maintain that obvious impossibilities do not have entailments while unobvious ones do, do we not thereby throw away our last chance of capturing the concept of entailment in abstract, highly general, person-neutral principles of entailment?

Another attempt: if A $\rightarrow$ B, then A 'gives a reason for' B; but obviously (P & $\neg$P) does not 'give a reason for' every Q. This value of φ is surprisingly popular. Someone who wishes to use it argumentatively ought, one would think, first to explain what he means by 'give a reason for' and then to test his principle when it is construed in accordance with the given explanation; but I have not yet found a devotee of the principle who is prepared to undertake even the preliminary, explanatory task. I am inclined to say that no obviously impossible proposition 'gives a reason for' anything at all; but that is because I understand 'giving a reason for' as a partly epistemological notion. Perhaps those who invoke it against Lewis intend it as purely logical? If so, then they owe us an account of what logical notion they take it to be. For example, they might explain 'A gives a reason for B' as meaning that A $\rightarrow$ B. That would secure their principle, all right, but for obvious reasons it would debar them from using it as one premise in an argument against the paradox.

Watling has argued that 'neither necessary nor contradictory propositions can be reasons for other propositions', and says that this inclines him to believe that necessary and impossible propositions do not entail anything.[19] By omitting the qualification 'obviously', he avoids the problem of drawing the obvious/unobvious line, only to run into the difficulty over argument by ***reductio ad absurdum***. His treatment of the latter connects entailment not only with 'reasons' but also with subjunctive conditionals (p. 146), and introduces the notion of 'conceiving' as well:

> I agree that we often speak of having reasons for, or against, necessary and contradictory propositions, but I do not think that these assertions should be taken literally, any more than I think that argument by ***reductio ad absurdum*** involves conceiving a contradiction to be true [p.148].

Watling goes on to remark candidly that 'I cannot give an explanation of how such assertions should be interpreted'; and I think it is fair to say that until explanations are offered, and the relevance of 'conceiving' made extremely clear, the case is too undetailed to be argued with.

It remains to be shown, then, that subjunctive conditionals or the notion of 'giving a reason for' can get us anywhere. While we are at this level of vague informality, though, I call attention to the following principle:

> A $\rightarrow$ B if and only if there is a route from A to B in which each move is licensed by our system of logical truths.

This is no cloudier than the other three principles I have been discussing; despite its greater vulnerability (it is a biconditional), it is at least as plausible as any of them; and it positively supports the paradox. For what the paradox says, when interpreted in accordance with the above principle, is that if you start with something conflicting with the system of move-licenses there is nothing you cannot arrive at. Looked at in this way, Lewis's thesis is very unsurprising.[20] The only surprise is that it should emerge so directly from a simple, beautiful analysis of the concept of

----

[19] Watling, op. cit., p. 149.
[20] Cf. C. I. Lewis, ***Survey of Symbolic Logic*** (Berkeley, 1918), p. 338.

entailment. It is, I suggest, a positive merit in Lewis's analysis that it implies that each impossible proposition entails every proposition.

XI. A REAL-LIFE EXAMPLE

One charge that I have heard leveled against the Lewis argument is that it is 'just a trick'. Certainly, it looks artificial and contrived, but then so does this:

> Let n_1, n_2, . . ., n_k be the set of all the primes up to and including n_k. Then consider the number $M = (n_l \times n_2 \times . . . \times n_k) + 1$. If M is divided by any one of n_1, . . . n_k there is a remainder of 1. So either M is divisible by some prime greater than n_k or else M, which is greater than n_k, is itself prime. Either way, there is a prime greater than n_k. So there is no greatest prime.

Like every elegant and purposeful deductive argument, this classic proof is 'a trick' in the sense that its moves are assembled with deliberate cunning in order to reach the desired conclusion. Is there any other sense in which the Lewis argument can be called 'a trick'?

Those who protest about 'tricks' may mean that nothing with the fundamental structure of the Lewis argument could occur in real life, as an ordinary human argument carried through innocently by someone who thought he was getting somewhere. Even if they were right about this, to conclude that the Lewis argument is therefore invalid would be to subject it to more strenuous demands than are usual when entailment principles are in question.

Anyway, they are wrong in their premise; and I want to show this, not just in order to meet their psychological needs but also as a way of introducing some further philosophical points. Consider the argument:

> P: Mary went out with a woman last night;
> and ¬P: Mary did not go out with a woman last night;
> so Q: Mary went out with a man last night,

which Lewis must deem to be valid. Perhaps it does not look valid; but if you wrote down an adequate set of axioms for arithmetic, followed immediately by 'So there are no n and m such that $(n/m)^2 = 2$', that would not look valid, either. One might even say that it was too condensed to be, humanly speaking, an argument; but the premises would entail the conclusion for all that. The argument about Mary, similarly, looks better when certain intermediate lemmas are supplied, such as those used in the abstract form of the Lewis argument. Here is a way in which the argument might innocently be carried through.

Helen knows that Mary went out with a woman. When I ask her, 'What did Mary do last night'? she replies, 'Mary went out with someone', thinking that I am merely concerned that Mary should not have been lonely in my absence. (Or I ask, 'Did Mary go out with someone last night'? and Helen answers, 'Yes'.) Wanting to hide my jealous suspicions, I take my next question to Jane: 'Did Mary go out with a woman last night'? Jane thinks that Mary stayed home, and so - answering in accordance with her belief - she says, 'No'. Having been told that Mary went out with someone, and that she did not go out with a woman, I conclude that she went out with a man (or a child - but that spoils the drama).

After the truth comes to light, I accuse Helen and Jane of having led me to believe that Mary went out with a man on the night in question; and each replies, truthfully, that what she told me was soundly based on what she believed, and that what she believed positively excluded Mary's having gone out with a man. Helen believed P and told me $(P \vee Q)$. Jane believed $(\neg P \,\&\, \neg Q)$ and told me ¬P. And I inferred Q.

No one has made a logical mistake here: not Helen, not Jane, not I. Or must we say that one of us erred logically because of what the others were up to? If so, then argument-validity depends upon where the arguer got his premises from, or on what someone else is going to do with his conclusion. That perhaps accords with the view that entailment is not transitive, or in a different way with the person-relative notion of 'intensional disjunction'; but it is a high price to pay for the 'intuitions' of those who refuse to accept what the Lewis argument so clearly shows. (Lewis himself shared those intuitions, and then rationally set them aside. Compare his first, Nelson-like paper on the topic with what he wrote only two years later.[21])

The argument about Mary is (barring the child) logically valid, and it might well be conducted in real life by people whose concern was not with the Lewis analysis but with Mary. I submit, further, that the logic of the Lewis argument can be displayed in indefinitely many other examples, whose validity is highlighted by their being possible - even plausible - slices of real argumentative life.

XII. SPECIAL FEATURES OF THE EXAMPLE

be the case that each example I could give was valid only because of special features not shared with every instance of the Lewis argument. Let us look at the special features of the argument about Mary.

First, the logically impossible premise is distributed between two people, one conjunct apiece, and what the conjuncts jointly entail is funneled through a single person only after the weakening has gone far enough to eliminate the impossibility. Slice-of-life instances of the Lewis argument need not be like this; but in a one-person example the premise has to be unobviously impossible if we are plausibly to suppose that the protagonist, though capable of conducting the argument, might fail to spot the impossibility. This requires either a wordy premise, or a long and complex argument, and such examples are therefore inconvenient; but they can be constructed.

I concede - if that is the word - that any plausible slice-of-life example of a Lewis-type argument must either split the premise between two or more people or have a wordy premise or a lengthy argument. But this requirement for plausibility is not a requirement for validity: if it were, we should have to say that obviously impossible propositions do not entail anything; and that thesis, as well as being hard to fit into any system of general principles of entailment, seems to have nothing to recommend it.

(I have been ignoring the distinction between 'The propositions $P_1, \ldots, P_n$ jointly entail Q' and 'The proposition $(P_1 \ \& \ldots \& \ P_n)$ entails Q'. Important as it is in purely formal work, it clearly has no place in Our present concerns.)

Second, the conclusion and the premise pair both concern what Mary did last night. This, I think, is the only other special feature of the example which needs discussion. Clearly it does need discussion: this topic overlap can plausibly be represented as a meaning-connection which the premise has with the conclusion but not with every proposition, and this might be thought to resurrect the issue which I claimed to dispose of in Section IX. So the topic overlap must be faced. If the example did not need this special feature, why did I include it? If the feature was required, then isn't Lewis in trouble?

----

[21] C. 1. Lewis, 'Implication and the Algebra of Logic', *Mind*, XXI (1912); 'The Calculus of Strict Implication', *Mind*, XXIII (1914).

It is clear that in any instance of the Lewis argument the conclusion will be topic-linked with the premise if and only if the disjoining move - the move from P to (P $\vee$ Q,) - introduces a disjunction whose disjuncts are topic-linked with one another. In the argument about Mary, the move from ‘Mary went out with a woman’ to ‘Mary went out with someone’ is what corresponds to the disjoining move in the abstract Lewis argument; and that way of describing it depends upon treating ‘Mary went out with someone’ as equivalent to ‘Mary went out with a woman or Mary went out with a man’ (we are still barring the child). These disjuncts are certainly connected or topic-linked, for they both concern whom Mary went out with last night; and that is precisely the topic overlap which, we noted, obtains between the premise pair and the conclusion. So my original questions give place to the following. If the ***disjoining move*** did not have to have that special feature, why did I include it? If the feature was required, isn’t Lewis in trouble?

The short answer is: it was necessary to introduce a disjunction with topic-linked disjuncts, but Lewis is not in trouble. The disjuncts had to be topic-linked if the example was to be plausible as a slice of real argumentative life, but this does not embarrass Lewis’s analysis because it is a requirement only for plausibility and not for validity. I now defend these claims.

XIII. TOPIC-LINKAGE

It is clear that a disjoining move can be plausible only if the person who makes it has some reason for moving from P to the weaker (P $\vee$ Q,). Broadly speaking, he will have such a reason if he has (or thinks that his audience has) some practical or theoretical interest which is about as well served by the information that (P $\vee$ Q,) as by the fuller information that P. For example, if my only concern is that Mary shall have had company, then ‘She went out with a woman or she went out with a man’ meets my need about as well as ‘She went out with a woman’. If what is in question is only Smith’s immunity to certain legal sanctions, then ‘Smith is a U.S. senator or Smith is a U.S. representative’ will serve as well as ‘Smith is a U.S. senator’.

I contend that one source of our inclination to say that a given P and Q are topic-linked, or are about the same thing, is their being such that we might in easily imaginable circumstances have interests which were as well served by (P $\vee$ Q) as by P alone. A case in which P and Q strike us as ‘having nothing to do with each other’ will be one where we cannot easily envisage having any use for (P $\vee$ Q) except as a premise in a disjunctive syllogism whose conclusion was just one of the disjuncts. I am not saying: when P is not topic-linked with Q, we (sensible people that we are) decline to have interests which would be as well served by (P $\vee$ Q) as by either disjunct. My claim goes in the opposite direction: when we cannot think of any interest of ours which would be as well served by (P $\vee$ Q) as by either disjunct, we .express this fact by saying that P and Q are not topic-linked, or have nothing to do with one another.

So a plausible disjoining move requires disjuncts which are in the indicated way ‘interest-linked’ for the arguer or for his intended audience. This requirement for plausibility or naturalness, however, cannot be a requirement for validity; for the question of whether a given disjoining move satisfies the ‘interest-linkage’ condition is a question about what the arguer believes and about what he wants to know or to achieve. Whatever ‘F’ and ‘G’ may stand for, someone can be plausibly supposed to move from Fa to (Fa $\vee$ Ga) just so long as there is some proposition P such that he thinks that (Fa $\vee$ Ga) gives him reason to think that P, he has a practical or theoretical want which is partly satisfied by the information that P, and he does not think that either Fa or Ga alone would give him reason to accept any Q which would satisfy his want better than P does. If that is right, then to treat interest-linkage as required not merely for plausibility but also for validity would be to put validity at the mercy of the personal interests and flatly contingent beliefs of the arguer concerned.

I have put the point in terms of a move from Fa to (Fa ∨ Ga), but it holds for all disjoining moves. It is easy to see how it works, for example, in the move from 'There is an F' to 'There is an F or there is a G'. The previous example might be thought suspect because both disjuncts concerned the nature of a; and the disjuncts of the present example may also be said to be topic-linked because each concerns *what there is*. This raises an issue about word overlaps, which I shall discuss shortly; but it may be noted that the notion of topic-linkage is now becoming pretty attenuated. (Is every pair of propositions topic-linked because each member concerns what is the case?) Anyway, we can go still further afield. The further we go - the less connected the disjuncts strike us as being - the less plausible the disjoining move will be, considered as a possible slice of our argumentative lives; but, however disparate and unrelated P and Q seem to us to be, we can always try to describe a possible person whose beliefs and interests are so peculiar that the move from P to (P ∨ Q) would be natural and pointful for him. I see no reason why we should ever fail except through lack of inventive talent - and Section XIV will show how little of that is needed.

There is a further point, which must be introduced with care. Wanting to show that plausibility requires topic-linkage but validity does not, I have introduced the notion of interest-linkage. Now, if I could maintain that all a disjoining move needs in order to be plausible is an interest-linked pair of disjuncts, and that all there is to our sense that a given P and Q are topic-linked is that they arc interest-linked for us, then my case would be complete; for this would amount to replacing 'topic-linked' by 'interest-linked', and the latter is rather clearly not a requirement for validity. But the case is not complete, for the following reason.

Someone who accepts P but has (as it were) no use for it which he cannot equally make of the weaker (P ∨ Q) will not just on that account replace P by (P ∨ Q). For he will not usually make that replacement - that disjoining move - unless he can express (P ∨ Q) about as briefly as he can express P. A fortiori, he will not ordinarily make the move if he has to express (P ∨ Q) in a sentence in which 'or' occurs between a P-expressing clause and a Q-expressing clause. Suppose that I ask Helen, 'What did Mary do last night'? She knows that Mary spent the evening playing bridge; she knows that I am anxious merely that Mary should have enjoyed her evening; and she knows (that I know) that Mary enjoys only evenings spent playing bridge or reading novels. Helen will not be likely to answer, 'Mary spent the evening playing bridge or Mary spent the evening reading a novel', or even, more briefly, 'Mary spent the evening playing bridge or reading a novel'; although these are the answers which, so far as content is concerned, most economically meet my known need. For the alternative answer, 'Mary spent the evening playing bridge' is much briefer still, and yet contains the needed reassurance in an easily extractable form. In general, then, plausible disjoining moves will involve some way of expressing (P ∨ Q) about as briefly as P alone, and thus without a clause-linking 'or' - as when 'She went out with someone' is used to say that she went out with a man or she went out with a woman, or when 'He is a member of the U.S. Congress' is used to say that he is a U.S. senator or he is a U.S. representative. I shall express this by saying that in a plausible disjoining move the disjuncts must be fusible.

In the Mary example, furthermore, the disjuncts are expressible in sentences beginning with the same five words, 'Mary went out with a . . .'. This is neither necessary nor logically sufficient for fusibility, but it is obviously part of the story about how these two disjuncts are fusible; and it also has a lot to do with our conviction that the disjuncts are topic-linked. In the bridge/novel case, for example, each disjunct can be expressed in a sentence beginning 'Mary spent the evening . . .': this makes them semi-fusible, so to speak, by making it possible to express the disjunction in the form ‘Mary spent the evening playing bridge or reading a novel’; and it gives excellent grounds for saying that the disjuncts are topic-linked because each concerns how Mary spent the evening.

So plausibility requires not just interest-linkage but also fusibility; and it is easy to see that the latter - especially when it involves word overlaps between the disjuncts - is part of what it is for a given P and Q to be topic-linked. To complete my case, then, I have to show that fusibility is not required for validity.

Suppose that it were. Suppose, that is, that the move from P to $(P \vee Q)$ is to be deemed valid only if $(P \vee Q)$ can be expressed about as briefly as either P or Q alone, and a fortiori can be expressed as something other than an ‘or’-containing brute-force disjunction. (The claim that the move is valid only if $[P \vee Q]$ is expressed in this way would, as I have implied in Section III above, require that this whole area of philosophical logic be reconstructed from the ground up.) Now, the requirement that a given P and Q be fusible - that is, that a given $(P \vee Q,)$ be capable of expression other than as a brute-force disjunction, can be taken in either of two ways.

(1) It could concern what resources are available within a given language at a given time: if the argument is expressed in English, then its disjoining move is valid only if there is a suitably brief English sentence which expresses the disjunction. If that were right, we should have to take seriously an episode in Paul Jennings’s skit on existentialism: ‘It therefore follows (or at least it does in the French) . . .’. Also, suppose that a disjoining move is condemned by this criterion, and that then someone invents some terminology which allows the disjunction to be expressed as briefly as either disjunct: does this innovation transform an invalid move into a valid one? If innovations do not count as part of the language until they gain public acceptance, then how many English-users must accept the new terminology for the disjoining move to be validated? What if the terminology which validates the disjoining move occurs in the biggest dictionaries but is understood by hardly anybody? It seems clear that this approach is doomed.

(2) Alternatively, the requirement could be this: a disjoining move is valid only if the language could contain a sentence which expressed the disjunction with suitable brevity. This, however, rules out nothing. There is no pair of propositions P, Q such that a language could not contain a way of expressing $(P \vee Q,)$ as briefly as its briefest way of expressing either disjunct alone. For a given P and Q, we may fail to see any good reason why any language should have such resources, and we may even think there could not be a good reason. Still, even if there could not be good reasons for a language’s being specially adapted to expressing $(P \vee Q)$ economically, there can always be bad reasons - that is, ones based on false beliefs about how the world hangs together. When we devise systems of entailment principles using Ps and Qs, we are presumably seeking results which are applicable also to languages which reflect peculiar interests and eccentric or wrong world-views.

I conclude that fusibility is as irrelevant to validity as is interest-linkage, that fusibility and interest-linkage exhaust the basis for our judgments as to topic-linkage, and that the latter is therefore irrelevant to validity. It may be noted in passing that my treatment of fusibility has referred to the possibility of certain linguistic structures which strike us as peculiar while my treatment of interest-linkage has referred to the possibility of certain theoretical or practical interests and beliefs which strike us as peculiar. This parallelism is not an accident, for fusibility and interest-linkage are connected with one another: the more interests we have which are as well served by $(P \vee Q)$ as by P alone, the more reason there is for our language to contain means for expressing $(P \vee Q)$ about as briefly as P alone.

XIV. RELEASE FROM THE SPECIAL FEATURES

I contend that any instance of the Lewis argument can be fleshed out into a plausible slice of life just so long as (a) either its premise is unobviously impossible or the conjuncts in its premise are suitably distributed between two or more people, and (b) its disjoining move is one which the arguer can be plausibly supposed to have some reason to make. If that claim is false, it ought to be easily refutable by counterexamples.

If it is true, and if I have been right in arguing that these two preconditions for plausibility are not requirements for validity, then the Lewis argument has passed a very stiff test indeed.

In my one real-life example - namely, the argument about Mary - the preconditions for plausibility were satisfied because (a) the conjuncts in the premise were distributed between two people, and (b) the disjoining move looked reasonable not only to the arguer but also to us. I now offer a second example in which the preconditions are satisfied in quite different ways. In this example, (a) only one arguer is involved. Also, (b) the disjuncts are not linked by our interests, are not fusible in our language, and are not naturally expressible in sentences with any significant word overlap; so that to us there seems to be no topic-linkage between the disjuncts and none, therefore, between premise and conclusion. Here is the story.

A certain Oracle has said, ‘There will be rain this month or the King will die’, and a tribesman is interested in this prediction, seriously wonders whether it is true, and expresses it to himself in the form: ‘The Oracle spoke truly’. He accepts the sound agricultural maxim that

(1) If there is rain this month, the harvest will be ruined,

and the unchallenged tribal dogma that

(2) If the harvest is ruined, the sky god will be angry.

It occurs to him that these two beliefs imply that

If the Oracle spoke truly, then either the sky god will be angry or the King will die.

‘If the Oracle spoke truly’, he mutters, ‘it’s going to be a bad month for someone; and the thought of the sky god’s anger is terrifying. Perhaps the Oracle didn’t speak truly but I daren’t depend on that, so I’ll take precautions. I know that

(3) If I sacrifice a goat, the sky god won’t be angry,

and although that is expensive it will be worth it, if only for my peace of mind’. He sacrifices a goat, and continues: ‘Now that

(4) I have sacrificed a goat

I can rest assured that

The sky god won’t be angry,

and what a relief that is’! A little later, rain comes spattering down, and the tribesman notes:

(5) ‘It is raining

and so, after all,

The Oracle did speak truly!

I am glad that I sacrificed that goat to placate the sky god! Still, wasn’t I satisfied that some disaster would occur if it turned out that the Oracle spoke truly? Yes, indeed! Since the Oracle did speak truly, it is settled that

Either the sky god will be angry or the King will die;

and I - fool that I am - have placated the sky god’! Whereupon he bitterly accuses himself of regicide.

The tribesman’s premises are numbered (1) to (5). The formal presentation of this argument, with the other indented propositions as lemmas, is left to the reader. The crucial point about the argument is that the tribesman reaches the conclusion that ***The King will die*** although it is not one of his premises nor even a constituent of any compound premise. The argument is aided by the availability of *The Oracle spoke truly*; but that is not a premise, or a constituent in any compound premise, either.

XV. THE SECOND LEWIS ARGUMENT

Lewis's analysis of the concept of entailment also implies that each necessary proposition is entailed by every proposition. For this, too, Lewis has an independent argument:

(1) Q
(1) → (2) (Q & P) ∨ (Q & ¬P)
(2) → (3) Q & (P ∨ ¬P)
(3) → (4) P ∨ ¬P.

Perhaps some necessary propositions neither are, nor are uncontroversially entailed by, anything of the form (P ∨ ¬P); but even if that is so the above argument's validity would imply a stronger result than most of Lewis's opponents can stomach.

This second 'paradox' relates quite simply to the first: it is its contrapositive. Relations between the first and second Lewis arguments, however, are more complex. Cutting a long story short, the only challenge to the second Lewis argument which would not take us back over ground already covered is the challenge to (1) → (2); and this is what I want to discuss.

It has been claimed that (2) is not entailed by (1) but is entailed by

A: Q & (P ∨ ¬P);

and if A were the argument's premise then there would be nothing 'counterintuitive' about its having (P ∨ ¬P) as its conclusion. This position denies the widely accepted view that if (Q & R) → S, and R is necessary, then Q → S; and anyone who takes it must be careful. As Lewis Carroll once showed, we cannot automatically declare Q → S to be false just because the derivation of S from Q rests upon, presupposes, or requires a necessary truth which is not a conjunct in Q; for that would commit us to denying every entailment statement.[22] For example, the derivation of (2) from A rests upon

B: A ⊃ (2);

and the derivation of (2) from A and B rests in turn upon

C: (A & B) ⊃ (2);

and so on, backward and outward. So someone who denies that (1) → (2) must say that necessary premises are sometimes but not always deletable without loss of validity; and this merits serious attention only if we are told how to distinguish deletable from undeletable necessary premises.

One possible suggestion is this. If a given necessary proposition functions in a given argument, it is deletable without loss of validity if and only if its function is that of a 'rule of inference'. The main trouble with this proposal - which has its roots in some work of Lewy's[23] - is the difficulty of making the requisite distinction between two ways in which a necessary proposition can function in an argument. Lewy's attempt to make the distinction (pp. 137-138) is, as I think he might now agree, unsatisfactory; and it seems to me highly unlikely that anyone can

----

[22] Lewis Carroll, 'What the Tortoise Said to Achilles', *Mind*, IV (1895).
[23] C. Lewy, 'Entailment', *Aristotelian Society Proceedings* supp. vol. XXXII (1958), 134, 138.

clarify the notions of ‘necessary proposition which functions in argument A’ and ‘necessary proposition which functions as a rule of inference in argument A’ without abolishing the line between them.

Even if the required line can be drawn, why should the thesis embodying it be accepted? I suggest that any thesis which would have us reject the move from Q to (Q & P) ∨ (Q. & ¬P) as invalid would *ipso facto* stand convicted of misrepresenting the common concept of entailment. When we take a premise and split it up into jointly exhaustive subpossibilities, are we arguing invalidly or ‘suppressing’ something which needs to be explicitly stated? Such a move, if fully spelled out in terms of ‘and’ and ‘or’ and ‘not’, may look odd because it is too explicit to be typical of informal arguments, even careful ones. I can see no reason to agree that such a move is downright invalid because it is not explicit enough.

(The thesis that necessary propositions are always deletable without loss of validity is pleasingly relevant to the view of Emch and Vredenduin, touched upon in Section I, that the first paradox holds not for every impossible proposition but for every proposition of the form (P & ¬P. Suppose (a) that necessary premises or conjuncts in premises are always deletable, and (b) that each proposition of the form (P & ¬P) entails every Q. Then consider any impossible proposition R, of whatever form you like. By (b), (R & ¬R) entails every Q. Since R is impossible, ¬R is necessary and thus, by (a), deletable without loss of validity. Therefore R entails every Q. In short, if necessary premises or conjuncts are always deletable without loss of validity, the Emch restriction of the paradox is not a restriction. I owe this proof to Smiley, who says that it was current in the fourteenth century.)

XVI. VALIDITY

Before moving on, I must look back. At the outset I introduced ‘entails’ by equating ‘P entails Q’ with, among other things, ‘“P, therefore Q” is a logically valid argument’. Although this equation is generally accepted in the literature (Geach being an exception[24]), it is perhaps not quite right. As I mentioned in Section XI: if no one could see that P entails Q unless many intermediate lemmas were inserted, the simple ‘P, therefore Q’ hardly counts as an argument at all. (Whereas if P entails Q so obviously that no lemmas are needed, ‘P, therefore Q’ is naturally regarded as an argument-step rather than an argument. There is arguably no such thing as a one-step argument.) And a sequence of propositions of which each one after the first is entailed by earlier ones, even if it qualifies as ‘an argument’, may be so unsatisfactory that many speakers will prefer not to call it ‘valid’. Since this restricted use of ‘valid’ has some currency, it needs to be mentioned; but it does not affect any of my central contentions.

We have seen - and shall see again - that Lewis’s opponents sometimes object that his analysis allows as valid certain arguments which are manifestly unsatisfactory. I prefer to meet such objections by distinguishing between valid arguments and satisfactory arguments. We clearly have some such distinction: we have the notion of an argument which, though completely logically in order, is too congested or unclear or tortuous or the like to be capable of helping anyone to get from the premise to the conclusion. Some people will say that such an argument is not valid, and in extreme cases that it is not an argument; but I choose to use ‘valid argument’ rather generously, counting any sequence of contingent propositions $P_1, \ldots, P_n$ as a valid

----

[24] Geach, op. cit., p. 169.

argument for P_n from the premise P_1 just so long as each of $P_2, \ldots, P_n$ is entailed by one or more earlier members of the sequence. This departure from a usage which has acceptance in some quarters is not a stacking of the cards in Lewis's favor. Rather, it is required if certain objections to Lewis are to be even prima facie relevant to the issue over entailment. The substantive point is this: given that 'entails' is definitionally tied to 'follows logically from' and the rest, it is not possible both to tie it also to 'valid' and to use the latter in ways which take into account those psychological features of perspicuity and so forth to which I have referred.

XVII. THE CASE FOR THE SECOND PARADOX

The function of necessary propositions as principles of inference - which was being discussed at the end of Section XV - is important. It provides a basis for the following argument, which rests neither on Lewis's analysis nor on the second Lewis argument, for saying that each necessary proposition is entailed by every proposition.

Corresponding to any necessary proposition P there will be an argument-move which, if it were challenged, could be relevantly defended by pointing to the necessity of P. Suppose that you produce an argument of which I challenge one move M, and that I you defend M as validated by the necessary proposition P. I may deny that P is necessary, in which case we must argue about *that*. At the other end of the scale, I may agree that P is necessary and withdraw my challenge altogether. Even if I agree that P is necessary, though, I may still complain that it is hard to see that M is valid, and say that I should have been helped by an explicit statement of P. But this is not to say that P's suppression renders M invalid; rather, it is on a par with the complaint that an argument is too compressed, that too many lemmas are omitted, for it to be humanly satisfactory. Am I entitled, while agreeing that P is necessary and does support M, to make the stronger complaint that M is downright invalid unless P is stated among the premises? Someone who thinks that I may be - that is, that validity may be lost by the deletion of necessarily true premises - must presumably be muddling validity with educational or heuristic satisfactoriness. He may say that his position is not a muddle requiring diagnosis, but a tenable thesis supported by arguments; but then he had better produce some arguments.

So there are reasons, independently of Lewis's material, for saying that a necessary proposition is entailed by every proposition. We may validly make any move M which is supported by a necessary proposition P; and if M is valid there seems to be no objection to our clarifying M by stating P, introducing it as a kind of underived lemma to which we help ourselves in order to show more clearly how the argument is moving. If M may be made without P's being a premise, then why should not the M-clarifying statement of P also be legitimate even though P is not a premise?

To say that a necessary proposition is entailed by every proposition is, then, a kind of understatement. The second Lewis argument shows that it is true; but underlying it is the stronger truth that a necessary proposition is entitled to crop up anywhere in an argument without being explicitly related to anything that has gone before.

'But if that is right, then someone who is conducting a deductive argument, with any subject matter, is entitled to spawn necessary truths at will.' He can legitimately introduce any necessary proposition P which reflects and clarifies the logic of his argument. If P underlies a move which he is making but which does not need to be clarified, he is being over-explicit; if P does not underlie any move he is making, he is being irrelevant; but in neither case is the validity of his argument affected.

All that concerns the case where a proof line expressing a necessary proposition occurs either idly or as a move-clarifier; and in arguments from contingent premises that is usually the only way in which necessary propositions explicitly occur. For such an argument to ***terminate*** in a necessary proposition is simply a sign of failure - as when a schoolboy wrestles with a pair of simultaneous equations and ends up with the conclusion that (a + a) = 2a.

The following reasonable objection has been made: ‘The ***introducibility*** of any necessary proposition at any stage in an argument without loss of validity is not the same as the ***deducibility*** of any necessary proposition from any premises.’ To give weight to this distinction between introducing and deducing we must, it seems to me, turn to arguments of a different kind from those discussed above. That is, we must leave arguments from contingent premises to contingent conclusions, and consider instead the decidedly minority activity of arguing from necessary premises and thus arguing only to necessary conclusions.

In these cases necessary propositions do not occur only as move-clarifying lemmas, for these are arguments which constitute proofs of necessary propositions. So they do raise questions of the form ‘Does this necessary proposition follow from what has gone before?’ - questions which are clearly not answered by the points adduced so far in this section.

How are such questions to be answered? If Lewis is right, then arguments purporting to prove necessary propositions cannot be sorted out into valid and invalid simply on the basis of whether each non-premise line is entailed by earlier lines. I accept this consequence, and invite those who do not to defend and exemplify a rival position.

When we want disciplined proofs of necessary propositions, with criteria of acceptability which do not concern a proof’s aptness for a given task in a given community, we turn to formal systems. In virtually all such systems each admissible move corresponds, given the intended interpretation of the system, to a valid entailment; but in no such system does the converse of this hold. For a move to count as admissible, it is always demanded that it conform to the system’s explicit, syntactical, typographically-stated transformation rules. By being thus niggardly in the moves that we allow, we are enabled to keep control of what we are doing, to say what does and what does not ‘follow from’ the premises. The fact that, in such a system, we can distinguish between premises from which a given necessary proposition does ‘follow’ and premises from which it does not tells not a whit against the second paradox; and the dismal failure of all attempts to draw the distinction outside formal systems tells strongly in favor of it.

Connected with these points there is an issue about propositional identity. Lewis is faced with the following dilemma: either (a) there is only one necessary proposition and one impossible proposition, or (b) it is not the case that $(P \rightarrow Q) \& (Q \rightarrow P)$ is sufficient for P = Q. Clearly, option (b) must be accepted: it will not do to say that Gödel proved that all sisters are female and did not prove anything else. But that is all right; for mutual entailment is not the only possible basis for a criterion of propositional identity, even where contingent propositions are concerned. A question of the form ‘Is P identical with Q?’ can arise only if there is a sentence [P] which means that P and a sentence [Q] which means that Q; and I take it that the question of whether P is identical with Q is the question of whether [P] means the same as [Q]. Such questions are controversial, but it is not controversial to deny that the only viable approach to them is through the single rule of thumb: [P] means the same as [Q] if and only if P entails and is entailed by Q.

XVIII. CONCLUSION

The problem of analyzing the concept of entailment - that is, of saying in fairly abstract terms something comprehensive and illuminating and true about it - has an unusual feature. It rarely happens that, the deeper one explores the philosophical aspects of such a problem, the more confirmation one finds for the technically most elegant and powerful solution; yet that is the case here. The anti-Lewis literature of the past forty years has been marked by an apparent determination - which must also be rare - to have the worst of both worlds: technical complications and philosophical unthoroughness. I agree with what Lewis once wrote to me: ‘There is no way of escaping from the paradoxes except by forgetting to ask yourself some question of logic, or repudiating the plainly indicated answer’.