

IDENTITY AND CARDINALITY: GEACH AND FREGE

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P. T. Geach, notoriously, holds the Relative Identity Thesis, according to which a meaningful judgment of identity is always, implicitly or explicitly, relative to some general term.

‘The same’ is a fragmentary expression, and has no significance unless we say or mean ‘the same X’, where ‘X’ represents a general term (what Frege calls a *Begriffswort* or *Begriffsausdruck*). (P. T. Geach, *Mental Acts* (London: Routledge and Kegan Paul, 1957), p. 69.

I maintain that it makes no sense to judge whether things are ‘the same’, or remain ‘the same’, unless we add or understand some general term - ‘the same F’. (P. T. Geach, *Reference and Generality*, third Edition (Ithaca, N.Y.: Cornell University Press, 1980), pp. 63f.

I am arguing for the thesis that identity is relative. When one says ‘x is identical with y’, this, I hold, is an incomplete expression; it is short for ‘x is the same A as y’, where ‘A’ represents some count noun understood from the context of utterance - or else, it is just a vague expression of a half-formed thought. (P. T. Geach, ‘Identity,’ *Review of Metaphysics* 21 (1967-8), p. 3.)

One of the ways Geach seeks to support this is by tying it to the well nigh universally admired Fregean thesis about cardinality.

Frege sees clearly that ‘one’ cannot significantly stand as a predicate of objects unless it is (at least understood as) attached to a general term; I am surprised he did not see that the like holds for the closely allied expression ‘the same’. (*Reference and Generality*, p. 64.)

Frege emphasized that ‘x is one’ is an incomplete way of saying ‘x is one A, a single A’, or else has no clear sense; since the connection of the concepts one and identity comes out just as much in the German ‘*ein and dasselbe*’ as in the English ‘*one and the same*’, it has always surprised me that Frege did not similarly maintain the parallel doctrine of relativized identity . . . (‘Identity’, p. 3)

. . . the thesis that identity is always relative to . . . a criterion seems to me a truism, like Frege’s connected thesis that a number is always relative to a *Begriff*. It is as nonsensical to speak of identification apart from identifying some *kind* of thing, as to speak of counting apart from counting some kind of thing. A numerical word demands completion with a count noun; similarly for ‘the same’ and ‘another’. (P. T. Geach, ‘Ontological Relativity and Relative Identity,’ in M. K. Munitz, ed. *Logic and Ontology* (New York: New York University Press, 1973), p. 289.)

In this paper we will look at Frege’s doctrine of cardinality and Geach’s Relative Identity Thesis, each in the light of the other.

I

Geach is certainly justified in claiming a close connection between cardinality and identity. To say that $x = y$ is to say that there is just one of ‘them’, and that x and y between ‘them’ only make one. Whereas to say that $x \neq y$ is to say that there are two of them, that between them they make two. Likewise to say that there are three oranges in the sack is to commit ourselves to a number of identity statements for example, (if a , b , and c are the three oranges in the sack), that $a \neq b$, b

$\neq c$, and $a \neq c$, and that for any y , if y is an orange in the sack, then either $y = a$ or $y = b$ or $y = c$. Finally, assuming that x exists and y exists, consider the conditional:

If x is not y , then x and y are two.

The antecedent and consequent of this are obviously tightly interlinked: it seems impossible that the former should be conceptually innocent while the latter is faulty; and so, as Geach says, Frege's condemnation of the consequent would seem to commit him to condemning the antecedent as well.

Hence if a judgment of cardinality is ineluctably tied to a general concept, it is reasonable to suppose that a judgment of identity would be also. But if they are in the same boat, does this mean that Frege sinks with Geach or that Geach stays afloat with Frege?

Before coming to grips with this crucial issue we must do better than Geach on the relationship of the two doctrines. The suggestion of the second group of quotations is that the Relative Identity Thesis just amounts to saying the same thing for identity that Frege said for cardinality. Indeed the second quote from 'Identity' speaks of the 'parallel doctrine of relativized identity.' But that is, at best, an overstatement. John Perry points out that Frege most assuredly did not adopt what would have been an exact parallel of the Relative Identity Thesis, viz., a Relative Cardinality Thesis.¹ Such a thesis would run as follows:

Cardinal numbers are 'incomplete expressions'; whenever we attach a number to whatever we attach numbers to, we are, explicitly or implicitly, supposing a general concept to be paired with the number. Thus we can't say that they are four, simpliciter; we must say that they are *four oranges* or *four seeds*, or whatever. 'Four' (or 'has the number four') does not constitute a complete predicate on its own; rather, it can form part of indefinitely many predicates each of which is formed by pairing it with a general term.

This is not Frege's doctrine. Instead of relativizing the numerical *predicate*, what Frege did was to shift the *subject* of numerical predication: he held that a statement of cardinality, rather than predicating anything of an object, individual, group, or heap, predicates 'having n instances' of a concept.²

Let us nail down this distinction, using one of Frege's examples:

If I place a pile of playing cards in his hands with the words 'Find the number of these' this does not tell him whether I wish to know the number of cards, or of complete packs of cards, or even say of honour cards at skat. To have given him the pile in his hands is not yet to have given him completely the object he is to investigate; I must add some further word - cards, or packs, or honours. (Ibid. pp. 28-9)

So numbers do not attach directly to piles; we must add some general term to get a determinate question or answer. So far this sounds like Geach. If Frege were to continue in a Geachian vein,

¹ John Perry, 'Relative Identity and Number,' *Canadian Journal of Philosophy* 8 (1978), pp. 1-15. We are indebted to this article for a number of insights into the topic of the present paper.

² Gottlob Frege, *The Foundations of Arithmetic*, trans. J. L. Austin, (Oxford: Basil Blackwell, 1968): 'If I say "Venus has 0 moons", there simply does not exist any moon or agglomeration of moons for anything to be asserted of; but what happens is that a property is assigned to the concept "moon of Venus", namely that of including nothing under it. If I say "the King's carriage is drawn by four horses", then I assign the number four to the concept "horse that draws the King's carriage"' (p. 59). 'The number 0 belongs to a concept, if the proposition that a does not fall under that concept is true universally, whatever a may be . . . The number $(n + 1)$ belongs to a concept F if there is an object a falling under F and such that the number n belongs to the concept "falling under F , but not a "' (p. 67).

however, he would hold that cardinality does attach to the pile, except that it is not cardinality simpliciter but cardinality with-respect-to-cards or with-respect-to-packs or whatever. He would say that the pile itself is two pack-wise and fifty-two card-wise. But instead of taking that line, Frege introduces the general term in the other way, using it to pick out the object of attribution. It is not the pile, or any other concrete entity, to which a number property of any kind is being attributed: rather, a number property is attributed to the concept *card in this pile* or the concept *pack in this pile*. In Frege's words: 'The content of a statement of number is an assertion about a concept' (Ibid. p. 59). This shows the inaccuracy of Geach's statement 'Frege sees clearly that 'one' cannot significantly stand as a predicate of objects unless it is (at least understood as) attached to a general term.' Frege did not regard 'one' as a predicate of objects under any conditions.³

Just as we have constructed a variant on Frege's doctrine of cardinality which does make it run parallel to the relative identity thesis, we could instead modify the latter so as to make it parallel to what Frege actually held about cardinality. Having become convinced that 'a = b' won't do as it stands and that a general concept must be lurking somewhere in the vicinity, Geach might, in closer emulation of Frege, have gone on to construe identity as a relation between concepts. Instead of requiring the form 'a is the same F as b' he might have opted for 'The concept *a which is F* is uniquely coextensive with the concept *b which is F*.'

Do these complications blunt the force of Geach's appeal to the close connection of identity and cardinality? We think not. It seems clear that for both topics we can move freely between the 'changing the subject' version and the 'relativizing the predicate' version, that the two versions are motivated by the same considerations, and that they accommodate the same range of data. Thus Geach can still ask: if we adopt one of these 'generality' theses for number, how can we refuse to adopt some generality thesis for identity?

II

We are agreeing with Geach that his relative identity thesis will sink or swim with Frege's doctrine of cardinality, so far as their negative aspects are concerned. As for their respective positive doctrines: although, as we shall see, Geach's has one grave implausibility that is not

³ Indeed, Frege did not regard numerals as predicates at all. 'In the proposition "the number 0 belongs to the concept F", 0 is only an element in the predicate (taking the concept F to be the real subject). For this reason I have avoided calling a number such as 0 or 1 or 2 a property of a concept. Precisely because it forms only an element in what is asserted, the individual number shows itself for what it is, a self-subsistent object" (op. cit., p. 68).

Geach has called our attention to the following passage on p. 40 of the same work. Frege begins by saying 'In isolation, however, it seems that "one" cannot be a predicate.' He then continues in a footnote: 'Usages do occur which appear to contradict this; but if we look more closely we shall find that some general term has to be supplied, or else that 'one' is not being used as a number word . . .'. Taken in isolation this does suggest that Frege holds the 'Relative Cardinality Thesis', that he takes the basic story about cardinality to be that a numeral combined with a general term serves as a predicate of objects. But this passage is taken from the earlier parts of the *Foundations*, where Frege is criticizing various inadequate views of number. When he comes to present his own view, from §45 on, he makes it abundantly clear that statements of cardinality are to be understood as specifying which number 'belongs to' a given concept.

matched by anything in Frege's, the two are alike enough in their main thrusts to make it hard for either to float unless the other does also. We shall argue for joint submersion.⁴

The first order of business is to identify the source of our resistance to the relative identity thesis. The deepest source would seem to be our ability to carry out singular reference. We not infrequently succeed in picking out particular items - physical objects, events, experiences, properties, persons, institutions - by the use of proper names, definite descriptions, and indexical expressions of various sorts. Given that we have succeeded in picking out something by the use of 'a' and in picking out something by the use of 'b' it is surely a complete determinate proposition that $a = b$, that is, it is surely either true or false that the item we have picked out with 'a' is the item we have picked out with 'b'; nor do we have to range a and b, covertly or overtly, under a common concept in order to form an identity proposition with a determinate truth-value. If a is the number 15 and b is Sally's new hat, it is clearly false that $a = b$, and no question 'Aren't the same what?' is left dangling. Perhaps any referent is thought of as an 'item' or 'entity' or 'thing' in the widest sense of these terms. But if sortals like these will satisfy Geach's requirements ('The number 15 is not the same entity as Sally's new hat') then his view is indistinguishable from the 'absolute identity' view.

Of course we cannot give an example of a true identity proposition the terms of which do not fall under a common concept. If $a = b$, then every predicate applicable to a is applicable to b. But the basic question is not whether there are applicable common concepts but whether such concepts must enter into the identity proposition if it is to have a determinate truth value - to embody a determinate 'thought' or content. And that we deny. Success in each singular reference is not only necessary but sufficient for determinateness of the proposition; nothing else is needed. If we really have succeeded in picking out something with 'a' and in picking out something with 'b,' then either that is the same item or it is not. We don't have to enrich the question to read 'same lamp,' 'same worm,' or 'same function' in order to generate a determinate issue. We take this to be a fundamental truth about reference. If there isn't a truth of the matter about whether $a = b$ then we haven't unambiguously picked out a single item with each of our referring expressions. Perhaps we neglected temporal parameters, so that 'Uncle John's car' fails to distinguish between the one he had this morning and the one he had this afternoon. Or perhaps we were making false uniqueness assumptions, so that 'Jim's son' fails to pick out a single person. An identity statement that involves such defective references will fail to express a unique proposition with a determinate truth-value.

In some cases of defective reference, an answer to the Geachian question 'Same what?' may help to remedy the defect. For example, you show someone a golden coin which you then melt down, using the gold to make a new coin which you show him on his next visit. 'Is this the same as what you showed me on my last visit?' he asks, and you might reply 'It's the same gold but not the same coin,' thus offering encouragement to the relative identity thesis. But your 'same F but

⁴ Our attack on Frege will be confined to his account of cardinality assignments, his way of construing statements as to how many so-and-so's there are. We shall lodge no objections to his view as to what a number is, or to other aspects of his philosophy of arithmetic. Moreover, even with respect to his account of cardinality statements, we do not confidently deny that all cardinality statements can be read the Fregean way; and in Section V we shall concede that it is to Frege's interpretation that we must look for an all-purpose canonical form for cardinality assignments. Our opposition to Frege is confined to denying the thesis that all cardinality statements *must* be construed as statements about concepts.

not same G' answer can be understood, harmlessly and conservatively, as implying this: 'I don't know whether your "this" was a reference to the gold or to the coin, so I don't know which of two questions you were asking. The answer to the question about the gold is Yes, and the answer to the other question is No.' Thus, the language of relative identity can be used to remove ambiguities of reference; but in the absence of such ambiguities determinate identity propositions can be had without help from a shared concept.

We do not deny that we must attend to what kinds of things we have on our hands if we are to determine the truth of an identity proposition. But what has to be avoided is the following sophistical argument. 'The procedures involved in determining whether number a is the same as number b are utterly different from those required to determine, for example, whether person a is the same as person b. This shows that our criteria of identity are specific to the kinds of items that are being identified, which implies that different identity relations are involved.' That is quite wrong. One relation is involved, namely identity - numerical identity - the weakest reflexive relation - the only relation that everything has to itself and nothing has to anything else. And the differences in 'procedures' reflect no variety of identity relations but only the variety of relata.

But then why shouldn't we say the same thing about cardinality? Aren't two or more successful singular references sufficient to set up a determinate cardinality question? Suppose we pick out some particular item by 'a,' one by 'b,' one by 'c,' one by 'd,' and one by 'e'. Can't we then go on to ask how many that is? And won't that have a determinate answer, assuming that our attempted reference was successful in each case? There is nothing in all this about understanding this question to be really 'How many Fs?' for any F more specific than 'item', or 'entity.' Indeed there may be no such F available. What if a is the greatest prime number less than 38, b is Jim's copy of *Rasselas*, c is yesterday's thunderstorm, d is President Reagan, and e is Syracuse University? But, again, if there is such an F, as there would be if a through e were all organisms, why should we suppose that this must come into a cardinality question or proposition in order that it have a determinate content? Why isn't 'a, b, c, d, and e, are four' a perfectly determinate proposition as it stands? Indeed, if we carried out a successful reference with each of the five terms, then the truth of that proposition simply hangs on whether exactly one of the identity propositions formed by taking the terms pairwise is true. Because successful singular reference is an adequate basis for identity propositions, it is also an adequate basis for cardinality propositions. Thus does Frege sink with Geach, rather than Geach floating with Frege.

III

It would be unrealistic to expect a Frege-Geachian to yield so quickly. A likely basis for a counter-attack is Frege's theory of reference, according to which an individual referring expression picks out its referent by way of a 'sense'. Ignoring refinements, we may think of the sense of a singular term as determining a uniquely exemplified property; the expression with that sense picks out the one and only entity that has that property. We now argue that an acceptance of Frege's theory would make a difference to the details of the argument but not to the final conclusion that Frege is wrong about cardinality and Geach about identity.

If Frege is wrong about reference, that makes it easier to say and to show that he is wrong about cardinality. If he is wrong about reference, then not all singular referring expressions work through senses; that is, they do not all fasten onto a referent by virtue of the fact that it uniquely exemplifies a certain property. Call a referring expression that does not work that way an 'irreducibly proper name'. Russell's 'logically proper names' and Kripkean proper names are

among the varieties of ‘irreducibly proper names’. Now take ‘a’, ‘b’, and ‘c’ to be irreducibly proper names and consider the question: ‘How many are a and b and c?’ There is nothing here that could possibly count as the general concept under which the counting is to be done; the question is well-formed and determinate, yet its only elements are logical concepts, the concept of number, and irreducibly proper names. The general concept demanded by Frege is conspicuously absent, unless we suppose it to be the concept expressed by ‘identical with a or with b or with c.’ That would be to maintain that the real logical form of ‘How many are a and b and c?’ is ‘How many instances are there of the concept *being identical with a or with b or with c?*’ It seems clear, however, that if that supposedly general concept suffices to meet the demands of Frege’s theory and of Geach’s, each theory is deprived of its intended thrust.

But if Frege’s theory of reference is correct (and that is a question on which we take no stand), it suggests a *prima facie* possible line of escape for Frege’s view about number statements. When the items being counted are all of a kind - people, or symphonies, or numbers etc. - the relevant general concept is easy to find; I had lunch with five people, Beethoven wrote nine symphonies, etc. But even when they are utterly heterogeneous (a number, a copy of a book, a storm, a man, a university) there is still a property under which they are being counted, the Fregean may say. In referring to each item we have picked it out through a property which it alone has, and so there is a property which is common to all and only those items, namely the property of having one of the original identifying properties. Thus, according to our Fregean, the boring but impeccable proposition that yesterday’s thunderstorm and President Reagan and Syracuse University are three finally turns out to have the form ‘There are three instances of the concept *YT or PR or SU*’, where the pairs of letters correspond to the properties through which the three items have been referred to. So even with such an extravagantly heterogeneous list as that, Frege’s theory of cardinality is vindicated.

So the argument goes. But is this a true vindication?

The first thing to notice is that the argument has no tendency to show that successful singular reference is not sufficient as a basis for cardinality propositions: on the contrary, it contends that general concepts are needed for cardinality *because* they are needed for singular reference. It follows that the argument cannot rescue Geach’s relative identity thesis, however much good it does for Frege’s doctrine about cardinality. It is of the essence of the Relative Identity Thesis that x may be the same F as y even if it is not the same G as y (for example, the same piece of gold as y but a different statue), and this does not even make *prima facie* sense unless in it ‘x’ and ‘y’ are supposed each to refer uniquely. Geach must therefore hold that even when successful reference has been achieved something more is needed to yield a determinate identity proposition; and that is not provided by the argument we have adduced on behalf of the beleaguered Fregean. This brings out an important difference between Geach’s doctrine and Frege’s. For it seems to be Frege’s own view - and not just a presupposition of the argument we have invented for him - that in a cardinality proposition a general concept is needed only because without it the units to be counted cannot be picked out; whereas Geach contends that a determinate identity proposition requires successful reference and also, *in addition*, a general concept. That part of the Relative Identity Thesis cannot be saved by any success that Frege’s more modest doctrine might have.

Let us turn now from Geach’s theory back to Frege’s. If Frege is right about how reference must happen, then any cardinality proposition contains the raw materials for a (perhaps disjunctive) general concept. But Frege is claiming more than that. He clearly holds that the general concept must be used if one is to think a determinate cardinality proposition: a definite

‘How many?’ question must be understood as ‘How many Fs?’ where F is some general term. But the argument from the senses of singular referring expressions does not establish that. Given the semantic assumptions, it shows that wherever one has expressed a determinate cardinality (identity) proposition one is thereby in a position to determine the relevant unit(s) by the use of some general concept; but it does *not* show that one always does so, much less that one must. The argument is quite compatible with the possibility that one simply sets up one’s units by carrying out singular references, and making no use of any common property in doing so. Of course, the argument’s semantic assumption requires that the sense of a referring expression is used to pick out the referent: a speaker who successfully refers must be credited with (perhaps implicitly) grasping and using its sense. But it doesn’t follow that when we set up a cardinality question by making a number of references *seriatim*, using a sense in each case, we are (even implicitly) using the disjunction of those senses in specifying what is to be counted. It may come as a complete surprise to us that such a disjunctive item is a common property of the numerees. We may have been expressing cardinality propositions for decades without the idea of a disjunctive property showing up anywhere in our thoughts or in our practice. When we have picked out each member of a class with a different referring expression, the disjunctive property is, so to say, available to us; but here as elsewhere we finite mortals do not avail ourselves of all our resources.

Thus the most that can be extracted from the argument from Fregean senses is that wherever a cardinality proposition is set up by the use of referring expressions, it could also be set up by the use of a common property, that is, that the first way presupposes the *possibility* of the second way. And so even on a Fregean theory of singular reference, although we are driven to what we might call an ‘availability’ form of a generality theory, we are free to reject the ‘actual use’ form embraced by Frege.

Similarly, the most that Frege’s theory of reference could salvage from the relative identity theory would be an ‘availability’ form of it. If all referring expressions have Fregean senses, the question whether $a = b$ *can* be put as a question whether a is the same F-or-G as b, but there is no reason to think that we have to put it this way, that is, no reason to suppose that if we are to think the question in a determinate fashion we must think it with the aid of a general concept.

Summing up: we have argued that successful singular reference is all that is needed for determinate identity and cardinality propositions. That refutes the part of Geach’s relative identity thesis which implies that general concepts are needed for some purpose over and above reference to the item(s) being identified. We were then left with a question addressed to Frege’s doctrine of cardinality, and to the safer and weaker relative identity thesis that would result if Geach lopped off the extra bit just criticized - the bit that says that general concepts are needed in addition to successful reference.⁵ The question is: can the need for reference itself be parlayed into a need, in thinking a determinate identity or cardinality proposition, to use some covering general concept? Does the need for successful reference imply that cardinality questions have to be thought in the form ‘How many Fs?’ and identity questions in the form ‘Is a the same F as b?’ The answer falls into two parts. If there are irreducibly proper names, then the answer is simply and obviously No. For then we can have questions of the form ‘How many are a and b?’ where there is no further general concept to do the work that Frege and Geach say needs to be done. If on the other hand all reference is through Fregean senses, a suitable general concept is always available, standing in

⁵ In this discussion we are taking for granted the minor surgery that was needed at the outset in order to produce a reasonable semblance of parallelism between the two theories.

the wings; but there is no reason to believe that this concept must always be used by someone who determinately thinks the cardinality or identity proposition.

We conclude that Geach is not entitled to draw comfort for his relative identity thesis from Frege's doctrine of cardinality. The former has one false element which goes far beyond anything in the latter. There remains a substantive part of Geach's thesis which does have a parallel in Frege's; and neither survives criticism.

IV

We may be asked: 'What, according to you, is the logical form of such a proposition as that a and b and c are three?' We have two possible responses - one safe and the other a little risky.

The safe one is to say that every cardinality statement of the form ' . . . are n' where the blank is filled by a list, is a monadic predication on a class. Frege could accept that as the logical form of every cardinality statement, but then he would differ from us in demanding that the class always be specified through a defining concept, whereas we are willing to settle for an enumeration of its members.

The 'predication on classes' answer lets us stand our ground while meeting the demand of Frege and others that each predicate have a determinate valency or -adicity: that answer says that each cardinality statement makes a *monadic* predication on a class. The riskier response denies that legitimate predicates must have a fixed valency. Several recent writers have contended that the demand for fixed valencies is a mere prejudice, and that there is nothing wrong with predicates whose valency is variable - multigrade relations such as ' . . . live together' as it occurs in 'John and Mary and Charles live together' and 'the Mortons live together.'⁶ If such predicates are admissible, then it is open to us to maintain that each predicate of the form ' . . . are n' can properly take any number of arguments from two upwards. Thus, for example, ' . . . are one' can function as dyadic ('Cicero and Tully are one') or as triadic ('England and Albion and Blighty are one'), and so on upwards; for there is nothing logically or syntactically wrong with ' N_1 and N_2 and . . . $N_{1,000,0001}$ are one', with a million different names being used, though of course the odds are against its being true.

V

We have attacked one of the most admired bits of philosophy of the past century, and we are conscious of our iconoclasm. But although we have pulled down the icon, we have not harmed the cathedral. That is, in arguing against Frege's view that the only determinate cardinality propositions are ones of the form 'There are so many Fs', we are not challenging the important idea that if there is to be a preferred all-purpose canonical way of expressing cardinality propositions it should be of that form. It will be easier to explain this if we first layout the alternative possible forms a cardinality proposition can take:

- (1) The concept F has n instances
- (2) $\hat{x}(Fx)$ has n members
- (3) $\{x, y, . . .\}$ has n members
- (4) x and y and . . . are n.

⁶ Adam Morton, 'Complex Individuals and Multigrade Relations,' *Nous* 9 (1975), pp. 309-318; Richard Grandy, 'Anadic Logic and English,' *Synthese* 32 (1976), pp. 395-402; Barry Taylor, 'Articulated Predication and Truth-Theory,' in B. Vermazen and M. Hintikka eds., *Festschrift for Donald Davidson*, forthcoming.

Frege stresses (1), but the spirit of his position is well enough caught by (2).⁷ We are attracted by (4), but our fundamental disagreement with Frege would remain if we settled for (3). So let us, for ease of discussion, take it that the issue concerns (2) versus (3).

We want a single all-purpose canonical form of cardinality statement. Which of the two is it to be? If it is (2), then there will sometimes be trouble in constructing the required F. It may have to be of the form 'is G or is H or is I . . .' and so on, disjunctively working through the 'senses' of our Fregean names for the members of the class. And if there are logically proper names with no 'senses,' F must sometimes take the form 'is identical with x or with y or with z . . .'.

But that, though contrived and artificial, is possible. If we opted for (3) as our all-purpose canonical form of cardinality statement, on the other hand, it would often be impossible to say what we wanted to say. Many classes whose cardinality interests us are unlistably large, and even with quite small ones we are usually unable to produce the lists. When we get as small as zero, the inability is absolute. There is no list-giving alternative to 'The class of F's has no members.' Thus, as between (2) on the one hand and (3) or (4) on the other, Frege's choice, (2), is the only possibility for a canonical form of cardinality statement.

We might add a word at this point concerning unit classes, about which Frege made heavy weather. Frege was impressed by the strangeness of statements of the form 'x is one', and tried to explain it through his general doctrine of cardinality which implies - quite wrongly - that 'x and y are two' is equally strange.⁸ The real point about 'x is one' is just that something needed for the sentence to *have a truth-value* suffices for it to be *true*, namely that its singular term succeeds in referring. The sentence therefore cannot inform. That implies that such statements as 'Gottlob Frege was one' are true but unsatisfactory. And it lets us explain why - Frege to the contrary - statements of the form 'x and y are two' are not similarly unsatisfactory. For such a statement to have a truth-value, each of its singular terms must succeed in referring; but for it to be true something more is needed, namely that they refer to distinct things. So the sentence can inform.

The case for giving canonical status to cardinality statements of the form 'There are so many Fs' or 'The class of Fs has so many members' has no analog in the case of identity. Even if the relative identity thesis were whittled down to the triply weakened claim that if we want a single all-purpose form of identity statement it had better be 'x is the same F as y', there is still no parallel reason to believe it. Nor any other reason; for identity, enumeration will do quite nicely as an all-purpose technique.

⁷ 'Assigning a number always goes along with naming a concept, not a group, an aggregate, or such-like things; and . . . if a group or aggregate is named, it is always determined by a concept, that is to say, by the properties an object must have in order to belong to the group . . .' Gottlob Frege, Preface to *Grundgesetze*, in P. Geach and M. Black eds., *Philosophical Writings of Gottlob Frege* (Oxford: Basil Blackwell, 1970), at p. 140.

⁸ *The Foundations of Arithmetic*, pp. 29-31. See also pages 40-41 where Frege says "'Solon was one" [is] not intelligible on its own taken in isolation' and 'We cannot say "Thales and Solon were one",' clearly implying that 'Thales and Solon were one' is not merely false but unintelligible on its own taken in isolation.