The Age and Size of the World

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Kant rejected two views about the world: that it is infinitely old and that it is infinitely large. But he failed to make himself clear. One cannot be sure what his point is about the infinite age and infinite size of the world, and I haven't found the commentators very helpful either (see Section 2 below). In this paper I present a thesis about what was really troubling Kant in regard to those infinities, and about what solution he proposed for his troubles.

1. The age argument

Kant thinks of the world's past as a series, and equates the world's being infinitely old with this series' having infinitely many members. I shall speak of the series of past events, using 'event' as a purely technical term to mean 'one minute's worth of world-history'. It could as well be a year's worth or a century's worth, just so long as it isn't construed as anything like: whatever happened in the past hour, whatever happened in the half-hour before that, whatever happened in the quarter-hour before that, and so on; for that series can have infinitely many members without taking us back as far as lunch-time. Also, of course, the members of the series of past events must not be allowed to overlap one another.

When Strawson discusses this matter he pretends that 'for as long as the world has existed, a clock has been ticking at regular intervals', and he then equates the world's age with the length of the series of past ticks.[1] His 'ticks' do exactly the same work as my 'events'.

Now, Kant argues like this. If the world never began, then it has been going on for ever, and the series of past events—past ticks of Strawson's clock—has infinitely many members. But:

> The infinity of a series consists in the fact that it can never be completed through successive synthesis. It thus follows that it is impossible for an infinite world-series to have passed away. (A 426)

That is Kant's argument—his presentation of the alleged conceptual difficulty in the idea that the world is infinitely old. The argument looks bad, because on the face of it it is open to an obvious objection. Kant says that 'the infinity of a series consists in the fact that it can never be completed through successive synthesis'—that is, through a one-by-one enumeration of its members—but that is just false. A series of the sort Kant has in mind must, if it is infinite, be open at one end; it cannot have both a first and a last member; and so the enumeration of its members, *if started*, 'can never be completed'. But such an enumeration could be completed

[1] P. F. Strawson, *The Bounds of Sense*, p. 176.

all the same, if it did not ever start but had been going on for ever.

Let 'T' name a known point in past time, say the moment when you began reading this paper. Then Kant's argument can be put thus: the series of events-before-T' was completed at T; events could in principle be counted as they occur; and so a counting or enumeration of the series of events-before-T could have been completed at T; and so the series of events-before-T' does not have infinitely many members. But—the obvious objection runs—that final step is not valid, or anyway Kant hasn't shown that it is. For he hasn't displayed any incoherence in the idea that at T someone, said 'T minus 0', and a minute earlier said 'T minus 1', and so on—through every event-before-T, there being no earliest such event.

2. Commentators on the age argument

Of the commentators who discuss this matter in books on Kant,[1] the earliest I have read is Caird, who seems content with Kant's argument and wholly unaware of the obvious objection to it. Kemp Smith rejects Kan's conclusion indignantly, but hasn't the patience to look carefully at the argument Kant uses. According to Kemp Smith, apparently, Kant's premiss is that 'we cannot comprehend how, from an infinitude that has no beginning, the present should ever have been reached', which I find unKantian and unintelligible. Kemp Smith seems to find it *true*, but says that it does not justify us in 'denying what by the very nature of time we are compelled to accept', namely that 'time is. . . infinite, alike in its past and in its future'. All that is unhelpful because sheerly irrelevant, which one can't often say about Kemp Smith. In Section 4 I shall introduce another of his remarks which is not irrelevant but deeply and precisely wrong.

Ewing does at least expound the obvious objection to Kant's argument. He says that it accuses Kant of a 'puerile fallacy', which seems to me a bit strong; and he then proceeds to defend Kant against the obvious objection; but the defence seems to be quite incoherent. Weldon's treatment of Kant's argument is rather cursory and, in my opinion, not nearly critical enough. Gottfried Martin's anxiety to see Kant's argument as an implied commentary on earlier philosophers leads him to misrepresent it to an extent that must be seen to be believed.[2]

Other writers, such as Benardete, expound Kant's argument faithfully and attack it with the obvious objection.[3] Strawson also expounds the obvious objection and seems to regard it as fatal:

> A temporal process both completed and infinite in duration appears to be impossible only on the assumption that it has a beginning. If. . . it is urged that we cannot conceive of a process of surveying which does not have a beginning, then we must inquire with what relevance and by what right the notion of surveying is introduced into the discussion at all.[4]

I wholly agree with this. But, apparently unlike Strawson,

[1] The first four I shall mention are (1) E. Caird, *The Philosophy of Kant*, p. 567; (2) N. Kemp Smith, *Commentary to Kant's Critique of Pure Reason*, p. 484; (3) A. C. Ewing, *Short Commentary on Kant's Critique of Pure Reason*, pp. 211–2; (4) T. D. Weldon, *Kant's Critique of Pure Reason*, 2nd. edn., pp. 205–6.

[2] 'The impossibility of an actual infinite rests in the last resort on the world being created by God and, as God-created, being a world that is finite throughout.' G. Martin, *Kant's Metaphysics and Theory of Science*, p. 50.

[3] José A. Benardete, *Infinity* (Oxford, 1964), pp. 121–2.

[4] Strawson, *op. cit.*, p. 177. The best discussion of the First Antinomy, apart from Strawson's, is to be found in C. D. Broad, 'Kant's Mathematical Antinomies', *Proceedings of the Aristotelian Society*, 1954–5.

I am sure that the notion of surveying *has* a right to be introduced into the discussion, and indeed given a crucial place in it. I shall defend this later.

3. The size argument

If the world is infinitely large, Kant thinks, then the thought of *the size of the world* must be the thought of *every member of a series of finite world-parts*—e.g. a series of non-overlapping cubic miles of world. The size of' something finite can be regarded as what Kant calls 'the magnitude of a quantum which is... given in intuition as within certain limits': one need not think of its size serially, because, being finite, it is the sort of thing that might in principle be perceived in its entirety all at once. But if a thing's size is infinite, then:

> its magnitude can be thought only through the synthesis of its parts, and the totality of such a quantum only through a synthesis that is brought to completion through repeated addition of unit to unit. (A 428)

The point is spelled out, perhaps helpfully, in a footnote:

> The concept of totality is in this case simply the [concept] of the completed synthesis of its parts; for, since we cannot obtain the concept from the [perception] of the whole—that being in this case impossible—we can apprehend it only through the synthesis of the parts viewed as carried, at least in idea, to the completion of the infinite.

It is easy to guess how the argument will run from that point. Kant will object to the idea of an infinitely large world for the same reason that he objects to the idea of an infinitely old world:

> In order, therefore, to think, as a whole, the world which fills all spaces, the successive synthesis of the parts of an infinite world must be viewed as completed, that is, an infinite time must be viewed as having elapsed in the enumeration of all co-existing things. This, however, is impossible.

This is a most peculiar argument. Does Kant assume that if the world infinitely large then the series of past events is infinite? If his argument depends on that, it surely fails. Perhaps he is assuming only that if world is infinitely large then the series of past events *could* be infinite, arguing from this that since the series of past events *cannot* be infinite the world is not infinitely large. But it is not at all clear how Kant proposes to justify the initial assumption. What, for example, can we make of following way of putting the point?

> Unlike time, space does not in itself constitute a series. Nevertheless the synthesis of the manifold parts of space, by means of which we apprehend space, is successive, taking place in time and containing a series. (A 412)

Granted, a region of space can be thought of serially, e.g. as some small region, plus a yard-thick shell around it, plus a yard-thick shell around that, and so on. Granted also, a large enough region of space *must be apprehended serially*, so that the actual exploration of it would 'take place in time and contain a series'. But how do we get from those concessions to Kant's view—if it is his view—that if an infinite series of operations cannot be completed then the world is not infinite in extent?

I think that Kant entirely fails in his attempt to present the difficulty about the world's size as a special case or upshot or corollary of the difficulty about the world's age. I shall later argue that the attempt should never have been made—that the problem which Kant does have about the world's size ought to have been allowed to stand on its own feet.

4. The scope of Kant's problem

Although Kant denies that the world can be infinitely old or large, he thinks that it cannot be finitely old or large either. (The mistakes his anti-finitism involves, e.g. the assumption that something of finite size must have a boundary, lie beyond the scope of this paper.) So in the area I am discussing he sees himself as having not merely two views but two problems, each expressible in the form: 'We have grounds for wanting to describe x as infinite, but there is a difficulty about using the concept of infinity in this way.'

We know that Kant thinks he has two such problems—two values of x—though he hasn't made clear why they are problems, i.e. what the difficulty is about applying the concept of infinity to the series of past events or to the series of cubic miles (say) of world. All we have is an obscure reduction of the size problem to the age problem, together with an obviously defective account of the latter. One might conclude that Kant has shown unwittingly but all too clearly that his age and size 'problems' are bogus. I think that would be wrong, though; and as a preliminary to showing that it would be wrong I want to consider the question—what other problems of this general form does Kant think that he has?

I have contended that Kant ought to have allowed the size problem to stand on its own feet, rather than trying to reduce it to the age problem. But I do not mean that each should be presented just as the problem of how the notion of infinity can be brought to bear on the empirical world. Weldon sees Kant in that light, saying that according to Kant

> the understanding can frame no concept of an infinite series of places or events as an actual empirical object, since nothing like that can be given in experience,[1]

as though Kant's objection were to empirical infinities as such. But that misrepresents him, for he distinguishes clearly—or at least loudly—between infinities which do and ones which don't involve a conceptual difficulty. As evidence for this, and against Weldon, consider the following remark of Kant's:

> Since the future is not the condition of our attaining to the present, it is a matter of entire indifference, in our comprehension of the latter, how we may think of future time, whether as coming to an end or as flowing on to infinity. (A 410)

There are two points here. One is that we don't have to raise the question of whether the series of future events is infinite, whereas Kant thinks that we are forced to speculate about the world's age and size. But the quoted passage also implies clearly enough, I think, that in Kant's view we can suppose that the series of future events is infinite without thereby encountering any conceptual obstacle. So Kemp Smith is wholly wrong when he says:

> Kant limits his problem to the past infinitude of time. The reason for this lies, of course, in the fact that he is concerned with the problem of creation. The limitation is, however, misleading.[2]

This implies that the trouble Kant finds in the infinity of the series of past events is equally present—and perhaps even that Kant knows that it is equally present—in the infinitude of the series of future events. This, I contend, is a damaging mistake.

But that doesn't explain what the line is between the infinities Kant finds troublesome and the ones he doesn't. All we know so far is that a past infinity is troublesome while a future one isn't. What Kant says is that troublesome

[1] Weldon, *op. cit.*, p. 206.
[2] Kemp Smith, *op. cit.*, p. 484.

infinities are precisely those that lie in the past or involve the thought of an infinity that lies in the past. He expresses this by saying that an infinite series is troublesome if and only if it is a series of *conditions*, which he also calls a *regressive* series; and he says that source of the difficulty is 'the. . . idea of the absolute totality of the series of conditions of any given [thing which is] conditioned', and he explicitly that this idea 'refers only to all *past* time' (A 412). So the idea of an infinitely large world, though ostensibly involving a series lying wholly in the present, can be represented as a source of difficulty only by being shown to involve, covertly, the thought of an infinite series of past operations. And we have seen how unconvincing is Kant's attempt make this move.

He would have been spared the need to make the attempt if, instead of (a) equating the troublesome/innocent line with the past/non-past line, he had (b) equated the troublesome/innocent line with the non-future/future line. For then he could treat a present infinity, such as the infinite size of the world now, as troublesome not because it covertly stretches into past but just because it doesn't lie wholly in the future. I don't think that (b) would clash with anything solid in Kant's discussion of these matters. It would conflict with some of his remarks about 'series of conditions' and about the related distinction between 'regressive' and 'progressive' series; but these Kantian technicalities are not handled so firmly and cogently that we are forced to abide by them. Nor does (b) conflict with any of Kant's examples; for his only example of a 'progressive' or untroublesome infinite series is, precisely, that of the infinite series of future times or future events.

So we can fairly safely pretend that Kant's basis is (b) rather than (a). I now proceed to argue that this pretence brings positive advantages.

5. Starting infinite tasks

Kant's approach to any empirical concept is dominated by his view that anything I can intelligibly say about the empirical world must be interpreted somehow in terms of what I could, in principle, discover for myself by my own observations. This is a sort of first-person phenomenalism which is embodied in, among other things, Kant's theory that our concepts are just tools for the orderly management of our sense-impressions. This raises a question about the concept of infinity: how can I have any legitimate use for that concept in application to the empirical world? what experience of mine could possibly require me to make any use of it? Can I even intelligibly suppose myself to have experiences which justified a use of it? Kant raises these doubts by suggesting that the past of an infinitely old world, like the size of an infinitely large one, is 'too great for the understanding'—i.e. so great that we can't have a concept of it. This starts to sound like Weldon and Kemp Smith, but I add something that they omit—namely, that the concept of an infinite *future* is not, even by Kant's standards, 'too great for the understanding'.

The point I am making has been interestingly developed in Fred Dretske's paper 'Counting to Infinity' (*Analysis* 1965.) Dretske contends that it is possible—or at any rate only medically impossible—that someone should count all the natural numbers. He argues that we can intelligibly suppose that someone counts to 100, say; and if we can intelligibly suppose that someone counts to n then we can intelligibly suppose that someone counts to (n + 1); and so it makes sense to suppose that someone has just begun to count, and is going to count every natural number. There will of course never be a time at which he has counted them all, but given any natural number a time will come when he will have counted *it*.

Dretske's conclusion seems to me absolutely right. If it chokes you, dilute it a little: say of our supposed counter not that he will count all the natural numbers but that he will count each natural number. The basic point is just that we can make sense of the idea of beginning on some task and never stopping. Similarly, we can make sense of the idea that we shall last for ever: many people believe that they will last for ever, and I can see no incoherence in this belief, merely falsity.

Since many people don't see why Dretske is right, I shall linger for a paragraph. The statement that I shall count all the natural numbers is expressed by

$$(\forall n)(\exists t)(n \in N \rightarrow \text{I count to n before t}) \qquad \text{(A)}$$

where t ranges over times and N is the set of natural numbers. Those who protest, against Dretske, that I couldn't ever *complete* the counting of all the natural numbers are implying that *I shall count all the natural numbers* is equivalent to

$$(\exists t)(\forall n)(n \in N \rightarrow \text{I count to n before t}) \qquad \text{(B)}$$

It does seem natural to think that if I shall count them all I shall eventually have counted them all; or to think that if it is true of each of them that I shall eventually have counted it, then I shall eventually have counted all of them. Let us try to express this natural assumption in quantificational terms. The difference between (A) and (B), as expressed by the order of the quantifiers, is that between a weaker and a stronger statement—like the difference between *Everyone has a friend* and *Someone is everybody's friend*. So we cannot derive (B) from (A) without adding further premisses about counting or priority or numbers or the like. The basic relevant fact about counting is that if I count to n before t then I count to every lower number before t, which is to say that

$$((\forall n)(\forall t)(n \in N \ \& \ \text{I count to n before t}) \rightarrow (\forall m)(m < n \rightarrow \text{I count to m before t})) \qquad \text{(C)}$$

But (A) and (C) together still don't yield (B). An addition which does permit the derivation of (B), and apparently the weakest one that will do the job in a manner relevant to our present theme, is

$$(\exists n)(n \in N \ \& \ (\forall m)(m \in N \rightarrow m \leq n)) \qquad \text{(D)}$$

which says that there is a highest natural number, i.e. that the set of natural numbers is finite. The derivation of (B) from (A), (C), (D) depends upon no extra assumptions about counting etc.: it goes through, quite formally, with 'count to... before...' replaced by an arbitrary two-place predicate. I conclude that those who say that I shan't count all the natural numbers because I shan't ever have counted them all are ignoring a distinction—namely that between (A) and (B)—which is usually negligible but which is important in just such contexts as Dretske's, where we don't have (D) because what is being counted is an infinite set.

6. Completing infinite tasks

So much for the statement that I shall perform an infinite task. What about the statement that I have performed an infinite task? Dretske says 'I'm not sure that this makes sense'. His doubts do not concern the abstract logic of the statement, which, he shows, mirrors the logic of the statement he finds untroublesome. In particular, just as someone who will count to infinity won't ever have finished, so someone who has counted from infinity wasn't ever not-yet-started. Someone might say: 'And *that* is why the supposition doesn't make sense: for a task which one doesn't ever start is a task on which one isn't ever engaged and which one can therefore never finish.' That is wrong. It assumes a principle which is valid for finite tasks but not for infinite ones—the logical points involved being exactly those displayed in the preceding paragraph.

Yet Dretske, like Kant, doubts the intelligibility of the supposition that one has completed an infinite task. The

source of this doubt presumably involves facts about what it is to be a person, or to perform a task, or live through an event, or the like. Elizabeth Anscombe tells me that Wittgenstein, for some purpose, once invited his hearers to imagine coming upon a man saying '. . . nine, five, one, four, one, three, phew!' and then announcing that he had just completed a backwards recital of the entire decimal expansion of π. The conversation might go on like this: 'All of it?' 'All of it.' 'When did you begin?' 'I didn't begin, of course. I have always been reciting the decimal expansion of π, until just a moment ago when I finished—thank God!' If someone claimed to be embarking on a forwards recital of π, we wouldn't believe him, but we could understand what he said: we can take in the idea of doing so much, then a bit more, and, however much he had done, always a bit more still. But the creepiness of Wittgenstein's story, like Dretske's hesitancy, suggests that there is a conceptual difficulty in the idea of someone's completing an infinite task upon which he has always been engaged. It isn't clear that this is intelligible to us as a possible state for a sentient being.

The view that it is *not* intelligible—'the Kantian view', for short—is fairly widespread, and I am inclined to accept it. If it contains any truth, I think it must be for reasons of the following sort. The notion of someone's having performed a series of operations—if '*someone*' is taken seriously—involves the notion of his remembering performing those operations, or knowing what it is like to have performed them, or in some way *possessing* that part of his past. How much I have done or undergone is a kind of measure of how much of me there is now. And so, to suppose that I had performed an infinite series of operations is to suppose myself to be, *now*, infinitely experienced, or endowed with an infinite stock of memories, or something of that kind. And it can plausibly be maintained that that cannot be supposed. In contrast with this, the supposition that I shall perform all of an infinite series of operations does not involve the idea of my possessing, now or at any future time, anything like an infinite stock of memories. This contrast arises from a fundamental asymmetry in sentient beings: they have more epistemic grasp of the past than of the future. I think that this is a necessary truth. If it is not, then the possible sentients who falsify it won't accept the Kantian view and won't see any force in Kant's discussion of the age and size of the world.

A backwards recital of the natural numbers or of the decimal expansion of π is mechanically generated by a rule, and so it arguably burdens the memory with nothing more than the rule. But that feature of Dretske's and Wittgenstein's examples is just an expository convenience. Our concern is with the Kantian notion of a sentient being's epistemic grip on the contingencies of his past experience; and if we aren't to drift away from that, and thus from what is philosophically interesting in this area, we must now think in terms of non-rule-generated tasks or biographies, in which each episode is a partly brute-fact addition, imposing at least some extra load on the memory. For the same reason, we can ignore the boring possibility that someone should have lived for ever but at no time have memories stretching further back than, say, 100 years.

So much for infinite past versus infinite future. As for infinite present: there is clearly no room for that notion while we are concerned with what one can envisage oneself as encountering in experience. For example, there can be no question of supposing oneself to know the world to be infinitely large because one has perceived it, all at once, in all its infinite extent. In any Kantian spelling-out of things in first-person phenomenalistic terms, each present

must be extremely thin: the only way to build up a thick story—e.g. one which gives work to the concept of infinity—is by stringing together a series of presents. These will stretch into either past or future, and so they are covered by the previous discussion.

Perhaps this is what Kant is getting at in his purported reduction of the world-size problem to the world-age problem. His point there may be that the world cannot be infinitely large if 'the world's size' has to be elucidated in terms of what one would have experienced by the time one had ransacked the entire world, together with the point that that kind of elucidation seems to be implied by Kantian phenomenalism. But it must be confessed that if that is Kant's thought, then he expresses it most unclearly.

The unclarity can be explained. Kant thinks that the conflict between finitism and infinitism creates a problem which can be solved by appealing to a theory of his—'transcendental idealism'—whose only intelligible component is precisely the phenomenalism I have been discussing; and he claims that his theory's ability to solve this problem is a powerful argument in its favour. But of course that argument is viciously circular if the theory is also required to create the problem in the first place; and I conjecture that this is one reason why Kant is less than candid, or less than clear, about phenomenalism's role in creating an objection to the thesis that the world is infinitely large. Phenomenalism is in fact also involved in creating the other side of the size-problem, i.e. in Kant's objections to the world's being only finite in size; but that lies far beyond my present scope.

The reasons I have given for the Kantian view are assailable. In particular, I have no adequate answer to the following objection:

> 'I can suppose myself to have an infinite stock of memories, so long as I think of them as possessed dispositionally—which after all is how we do possess most of our knowledge of all kinds. My life so far has given me an accumulation of memories which are registered in me now as my ability to answer many questions about my past. The supposition that Kant thinks I cannot make is just that I should now be able to give—should now dispositionally know—the right answers to infinitely many distinct questions about my past. But of course I can suppose this. I can suppose myself able to answer five questions; and if I can suppose myself able to answer n then I can suppose myself able to answer (n + 1). So the Kantian view is false—or at least your defence of it doesn't work.'

One possible reply, anticipating a line of thought which I shall exploit in Section 10 below, is that the objection takes my epistemic possession of an infinite past to consist in my having certain abilities, i.e. in a fact about myself in a possible future. Furthermore, the objection had to do this. Memories may be episodic, occurring as states of consciousness whose relation to one's past is logically similar to that between one's sensory states and one's present objective environment. What cannot be supposed is that one should at any time have infinitely many memories of that kind. Any infinite stock of memories must be mostly dispositional, and so the infinity it involves must be in a certain sense projected into the future.

That reply, though I think it has some force, is less than compelling. I hope one can do better for the Kantian view than I have so far succeeded in doing. But my exegetical purposes don't require me to defend the Kantian view. It suffices that it is a view which Kant was inclined to hold, even if he didn't quite bring it to the level of consciousness. Given that much, I can explain some aspects of his thought

which in the literature that I have read have been left as mysteries.

7. The official solution

Before showing what really goes on in Kant's problem-solving endeavours regarding non-future infinities, I must sketch what he says is going on. On the face of it his official 'solution' is no solution at all, not even a bad one, but merely an inert piece of dogmatising. It turns out, though, to be a cover for two quite different problem-solving moves. One of these is mistaken, but they both have life in them. The relation between them cannot be properly understood except on the basis of a grasp of how each shelters under the dead 'official solution', to which I now turn.

According to Kant, a way out of the impasse is opened up by the realization that the world 'does not exist in itself', a claim which can also be expressed in Kantian language by saying that the world is a *phenomenon*. Kant's view is that a phenomenal item, but no other sort of item, can avoid being either finite or infinite, and so he is entitled to say: 'I. . . deny the existence of an infinite world, without affirming in its place a finite world' (A 503). 'If we regard the two propositions, that the world is infinite in magnitude and that it is finite in magnitude, as contradictory opposites, we are assuming that the world. . . is a thing in itself. . . ' (A 504). But it is not a thing in itself, and so we need not opt for either proposition.

When Kant says that the world is not a thing in itself, he means two sorts of things. **(a)** We are trapped on this side of the veil of perception: we cannot know 'things as they are in themselves' but only 'things as they appear to us', and so the world we know is only an assemblage of 'things as they appear'. **(b)** All our concepts are tools for the intellectual handling of our sensory intake: we cannot make sense of any statements about the world except ones admitting of a broadly phenomenalist analysis. Kant often has in mind both **(a)** and **(b)**, regarding them as parts of a single doctrine called 'transcendental idealism'. But in fact **(a)** is condemned by **(b)**: on Kant's own theory of what our concepts are, all our thinking is restricted to thoughts about actual or possible data, items which could be given or presented or made to appear to us; and so we cannot make sense of the notion of 'things as they are in themselves', i.e. the notion of something considered as having a certain nature which is not to be grasped or elucidated in terms of how the thing might appear to us. I shan't expand on this point, as I have already done so in my book, as has Strawson in his.

Kant too often discusses infinity in the spirit of **(a)** rather than **(b)**, implying that if the world were radically 'out there', beyond the veil, it might be infinite, but that since it is only an 'appearance' it cannot be infinite even if it isn't finite either. Such remarks, taken just as they stand, seem to me to be worthless—not just false, but dead.

If we are to salvage anything from Kant's use of the notion of 'things in themselves', as it occurs in the context of his infinity problems, we shall have to stress **(b)** rather than **(a)**. Roughly, we shall have to construe Kant as saying that a certain difficulty about non-future infinities can be removed by taking a phenomenalist approach to statements about the world, e.g. about its age and size. I believe that we can construe him thus. In the material that Kant presents us with, there are two strands which could be expressed in the form: 'Since statements about the world are to be understood phenomenalistically, the problem about non-future empirical infinities can be solved as follows. . . '. One strand maintains that phenomenalism shows us how the world can be neither finite nor infinite, whereas the other

maintains that phenomenalism shows us why it is not after all objectionable to suppose that the world is infinitely large and infinitely old.

8. The 'weakening' move

Of Kant's two purportedly problem-solving moves, I take first the one which doesn't work. I shall expound it in connection with the world's size: its re-application to the world's age is a routine matter, as will eventually become clear.

According to Kant's phenomenalism, any statement about the world is equivalent to a statement about actual and possible experiences. More specifically, any statement about how large the world is is equivalent to a statement about how long a series of experiences one could have, each consisting in the exploration of a hitherto-unexplored stretch of the world. It is very important that for Kant these are genuine *equivalences*:

> Only by reference to the magnitude of the empirical regress [i.e. the series of possible explorations] am I in a position to make for myself a concept of the magnitude of the world. (A 519)

That the series of possible explorations has such-and-such a length is not just a consequence of the world's having a certain size—it *is* the world's having that size.

So the statement that the world is not finite in size is to be analysed into the statement that the series of possible non-repetitive world-explorations has no end, i.e. that no finite series of explorations would exhaust the world, or that any finite series of explorations would leave some world unexplored. This has two different sorts of significance for Kant. The one that concerns me in the present section really has nothing to do with the notions of experience, exploration, 'empirical regress' etc. Abstracting from all such notions, we have Kant expressing 'The world is not finite in size' in the form 'No finite amount of world includes all the world there is' or 'Every finite quantity of world excludes some world'. This, I submit, seems to Kant to be a weaker statement than the statement that there is an infinite amount of world.

More generally, I am suggesting that Kant is one of those who think that

> Every finite set of Fs excludes at least one F, (1)

though it contradicts the statement that there are only finitely many Fs, is nevertheless weaker than

> There is an infinite number of Fs. (2)

Since (1) is weaker than (2), I think Kant thinks, the series of possible explorations can be more than finite without being infinite; and since the length of that series defines the size of the world, the world can escape being finite without being infinite.

I conjecture that Kant's reason for thinking that (1) is weaker than (2) is as follows. (1) is true if the Fs are the natural numbers, or the odd numbers, or the prime numbers, or the natural numbers > 7, or. . . ; but if (1) = (2) then each of these sets has an infinite number of members, and so they all have the same number of members. Kant can be forgiven for assuming that there cannot be exactly as many prime numbers as odd numbers.

The assumption is of course a mistake, even if a forgivable one. We now know that by the only viable criterion of equal-numberedness there are as many primes as odd numbers. If this seems 'counter-intuitive', that is presumably because our intuitions about cardinality have been fed almost exclusively by our thinking about finite sets.[1] Kant, for one, carries finitist assumptions over into his thinking

[1] Cf. Bertrand Russell, 'Mathematical Infinity', *Mind* 1958.

about infinity. He assumes that an infinite number can count as an honest-to-God number only if it is 'determinate'; and, though 'determinate' is not explained, it seems fairly clear that for Kant a determinate number is one such that if you add one to it you get a different number. From this it follows that a 'determinate' number must be a finite one, i.e. that there cannot be an infinite number. I doubt if Kant sees this consequence of his assumptions. In a footnote he refers to 'a quantity (of given units) which is greater than any number' and says that this 'is the mathematical concept of the infinite' (A 432): this seems to imply that there cannot be an infinite number, but I am not sure how seriously to take this.

A significant importation of finitist thinking into a discussion of infinity occurs in a passage where Kant congratulates himself for not using a certain bad argument against the world's being infinitely old or large. The argument he didn't use is this:

> A magnitude is infinite if a greater than itself, as determined by the multiplicity of given units which it contains, is not possible. Now no multiplicity is the greatest, since one or more units can always be added to it. Consequently an infinite given magnitude. . . is impossible. (A 430)

The middle sentence is wrong, because adding 'one or more units' to an infinite number does not yield a higher number; but Kant voices no objection to this finitist intrusion. He quarrels only with the argument's first premiss:

> The above concept is not adequate to what we mean by an infinite whole. It does not represent *how great* it is, and consequently is not the concept of a *maximum*. (A 430–2)

This complaint that the proffered definition of 'infinite' does not 'represent *how great* it is' seems to mean that the definition doesn't define a determinate number—one which is just so large and no larger, this being thought of as the notion of a number n such that n < (n + 1), and thus as the notion of a finite number. In short, having stayed silent on the argument's finitist error, Kant criticizes it on the basis of a finitist error of his own.

Kant is not the only philosopher to demand of infinite numbers a 'determinateness' which only finite numbers can have. Descartes says that 'in counting I cannot reach a highest of all numbers, and hence recognise that in enumeration there is something that exceeds my powers', from which he infers that 'a number is thinkable, that is higher than any that can ever be thought by me'.[1] Leibniz, too, shares Kant's nervousness about 'infinite number':

> It is true that there is an infinity of things, i.e. that there are always more of them than can be assigned. But there is no infinite number, neither of line nor of other infinite quantity, if these are understood as veritable wholes. . . The true infinite exists, strictly speaking, only in the absolute which is anterior to all composition, and is not formed by the addition of parts.[2]

There is also a nice example of the same line of thought in Locke:

> We have, it is true, a clear idea of division, as often as we think of it; but thereby we have no more a clear idea of infinite parts in matter, than we have a clear idea of an infinite number, by being able still to add new numbers to any assigned numbers we have: endless divisibility giving us no more a clear and

[1] Descartes, in Haldane and Ross (eds.), *Philosophical Works of Descartes*, vol. 2., pp. 37–38.

[2] Leibniz, *New Essays* II.xvii.1.

> distinct idea of actually infinite parts, than endless addibility (if I may so speak) gives us a clear and distinct idea of an actually infinite number: they both being only in a power still of increasing the number, be it already as great as it will. So that of what remains to be added (wherein consists the infinity) we have but an obscure, imperfect, and confused idea.[1]

I suspect that many other examples could be given, though these are all I have found so far. I should like to see a history of this matter. For example, it has been known since ancient times that every finite set of prime numbers excludes at least one prime number: I'd like to know when, and by whom, this was regarded as falling short of a proof that there is an infinite number of prime numbers.

9. 'Prior to all regress'

How does this spurious 'weakening move' of Kant's relate to his official solution? Part of the answer is obvious. The official solution says that the world is not a thing in itself, which I am construing as an assertion of phenomenalism. This leads Kant to equate *the world's size* with *the length of the series of possible world-searches*, so that 'the world is not finitely large' becomes equated with 'Every finite world-search leaves some world unsearched'; and this has nested within it the supposedly weaker-than-infinity statement that no finite amount of world includes all the world there is.

This link between the official solution and the 'weakening move' is an accidental one. The thought that 'Every finite world-stretch excludes some world' conflicts with 'The world is finite' without entailing 'The world is infinite'—this thought might have occurred to Kant in just that form, without his being led to 'Every finite world-stretch excludes some world' through its being nested within the phenomenalistic 'Every finite world search leaves some world unsearched'.

Still, the official solution connects with the 'weakening move' in another way as well, for Kant has certain formulations which can express both the thesis that the world's extent is not infinite and the pseudo-thesis that the world is not a thing existing in itself. Here is a crucial passage:

> We must not regard the world as a thing given in and by itself, prior to all regress. We must therefore say that the number of parts in a given appearance is in itself neither finite nor infinite. For an appearance is not something existing in itself, and its parts are first given in and through the regress of the decomposing synthesis, a regress which is never given in absolute completeness, either as finite or as infinite. (A 505)

Kant is there concerned with infinite divisibility (the 'decomposing synthesis') but the passage also bears directly on my present topic. When Kant says that the world isn't given 'prior to all regress', he means: statements about possible experience are not mere consequences of independently intelligible facts about the world, but rather give to statements about the world all the content we can understand them as having. But combined with that thought there is also the following different one. The statement that no finite set exhausts the members of a given series mustn't be thought of as a consequence of the series' having a determinate infinite number of members; rather, the statement that no finite set exhausts the series *is* the strongest statement we can make about the size of the series.

I think that both elements are present in this passage:

> The cosmic series can. . . be neither greater nor smaller than the possible empirical regress upon which alone

[1] Locke, *Essay* II.xxix.16.

> its concept rests. And since this regress can yield neither a determinate infinite nor a determinate finite. . . , it is evident that the magnitude of world can be taken neither as finite nor as infinite. The regress, through which it represented, allows of neither alternative. (A 518 n.)

I suggest that Kant is here playing with two thoughts at once, both having the form 'Our only concept of x is our concept of our approach to x'. One thought is that our only concept of the world is that of our actual and possible experiences of the world, while the other thought is that our only concept of a non-finite series is the concept of a series some of which always lies ahead of us.

The most striking example of all is in the following passage:

> Of this empirical regress the most that we can ever know is that from every given member of the series of conditions we have always still to advance empirically to a higher and more remote member. The magnitude of the whole of appearances is not thereby determined in any absolute manner; and we cannot therefore say that this regress proceeds to infinity. In doing so we should be anticipating members which the regress has not yet reached, representing their number as so great that no empirical synthesis could attain thereto, and so should be determining the magnitude of the world (although only negatively) prior to the regress—which is impossible. (A 519)

The mistake of 'representing the world's magnitude as so great that no empirical synthesis could attain thereto' is really two mistakes: thinking that questions about the world's magnitude concern something more than actual and possible experience; and thinking that we could say something about the world's magnitude stronger than that any finite stretch of world excludes some world. These are indeed both mistakes. But they have no direct and straightforward connection with one another. Kant thinks that they have, and indeed tends to identify them with one another, only because he has been misled by the protean phrase 'prior to the regress' and others like it.

10. The futurizing move

There is a second major element in Kant's purportedly problem-solving material—an element which more directly involves his phenomenalism. In exhibiting it, I take my stand on Sections 4–6 above: the infinities Kant finds troublesome are all and only those which don't lie in the future.

Let us look back at the phenomenalizing move as applied to the size of the world. The statement that the world is more than finitely large is equated with the statement that however much world I explore there will always be more world to be explored. Kant thinks that this stops short of saying that the world is infinitely large; but even if he didn't think this, and accordingly said that the world is infinite in extent, he would still be left with a problem-solving remark to make—namely that the relevant infinity is now projected into the future. This draws the sting from 'The world is infinitely large', because the latter is now equated with a statement about a possible *future* infinite series. According to the phenomenalist analysis, my thought of the world as being infinitely large is my thought of embarking upon a never-ending non-repetitive series of world-explorations—such a series being, though infinite, conceptually harmless because lying wholly in a possible future.

So much for the infinity of the world's present extent; and the same pattern of problem-solution applies even to the most completed-seeming infinity of all, namely the infinite series of past events. For if we take the statement that this

series is infinite, and ask about it the question that causes all the trouble—namely 'What does this statement mean in terms of what I could discover for myself?'—we find that the answer projects even the series of past events into the future, or into a possible future. That is, the idea of the series of past events is the idea of what I should discover if (in the future) I pursued my researches deeper and deeper into the past. Thus Kant:

> The real things of past time. . . are objects for me and real in past time only in so far as I represent to myself (either by the light of history or by the guiding clues of causes and effects) that a regressive series of possible perceptions in accordance with empirical laws, in a word, that the course of the world conducts us to a past time-series as condition of the present time—a series which, however, can be represented as actual not in itself but only in the connection of a possible experience. Accordingly, all events which have taken place in the immense periods that have preceded my own existence mean really nothing but the possibility of extending the chain of experience from the present perception back to the conditions which determine this perception in respect of time. (A 495)

In that passage, I think, Kant is making problem-solving remarks without properly grasping just what it is in them which solves the problem. This is hardly surprising since, as we have seen, he does not even succeed in explaining clearly what the problem is. There are other passages, too, where Kant subjects the troublesome infinity-statements to operations which project the relevant infinities into the future, and then relaxes. Here are two more examples (emphases mine):

> If I represent to myself all existing objects of the senses in all time and in all places, I do not set them in space and time [as being there] prior to experience. This representation is nothing but the thought of a possible experience in its absolute completeness. Since the objects are nothing but mere representations, only in such a possible experience are they given. To say that they exist prior to all my experience is only to assert that they are *to be met with* if, starting from perception, I *advance* to that part of experience to which they belong. (A 495-6)
>
> To call an appearance a real thing prior to our perceiving it either means that in the *advance* of experience we must meet with such a perception, or it means nothing at all. (A 493)

It is true that Kant does not explicitly set up his problem as one about non-future infinities as such; and it is also true that many of his remarks imply that there can be no legitimate use of the concept of infinity—indeed that there really isn't any such concept. But I still contend that a good part of his sense of having solved his problem is due to the fact that he is troubled about non-future infinities as such and the fact that he can see how, through phenomenalist analyses, to throw these infinities into the future.