

# Metaphysical Foundations of Natural Science

Immanuel Kant

1786

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[Brackets] enclose editorial explanations. Small ·dots· enclose material that has been added, but can be read as though it were part of the original text. Occasional •bullets, and also indenting of passages that are not quotations, are meant as aids to grasping the structure of a sentence or a thought. Every four-point ellipsis . . . . indicates the omission of a brief passage that seems to present more difficulty than it is worth. Longer omissions are reported on, between [brackets], in normal-sized type. Numerals in the margins refer to the pages in the Akademie edition of the work; these numbers are also supplied in both the existing English translations, which can thus easily be correlated with the present version.

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## Chapter 3

### Metaphysical Foundations of Mechanics

#### Definition 1

536 **Matter** is what can be moved, considered as having—just because it can be moved—a moving force.

#### Remark

This is the third definition of matter. The merely dynamical concept is different from this because it applies also to matter that is motionless. The moving force that was in question back there concerned merely the *filling* of a certain region, and we weren't permitted to regard the matter that filled the space as itself moving. So repulsion was a basic moving force for imparting motion, whereas in mechanics a force is regarded as actually at work in one portion of matter imparting motion to another portion. Very briefly and schematically: the movement of portions of matter is considered as a potential in dynamics, and as actual in mechanics. Clearly, a portion of matter won't have

- the power to make other things move by its own motion

unless it has

- basic moving forces through which it is active in every place where it exists,

this being an *activeness* that comes before any proper *motion*.

Breaking that point down into its two constituents: Clearly a portion of matter moving in a straight line and encountering another portion won't make the other move unless both of them have basic forces [Kant wrote *Gesetze* = 'laws', presumably a slip] of repulsion; and a portion of matter couldn't in moving drag another portion after it unless they both had attractive forces [*Kräfte* = 'forces']. So all mechanical laws presuppose

537 dynamical ones. . . . You'll notice that I shan't say anything

more about the communication of motion by attraction (such as might happen if a comet with a stronger attractive power than the earth came close to the earth and dragged it out of its orbit). I'll be talking only about the agency of repelling forces—i.e. agency by pressure (as by means of tensed springs) or by impact. I'll do this because applying the laws of repulsion is exactly the same as applying the laws of attraction except for the difference in direction.

#### Definition 2

The **amount<sub>q</sub> of matter** there is in a certain space is given by how many movable parts there are there. When this matter is thought of as having all its parts in motion at once, is called the mass; when all the parts of a portion of matter move in the same direction, exercising their moving force externally, the portion is said to **act in mass**. A mass with a determinate shape is called a **body** (in the mechanical sense). Mechanically estimated, the amount of motion is a function of two variables, namely: how much<sub>q</sub> matter is moved and how fast. . . .

#### Proposition 1

The only way of comparing the amounts<sub>q</sub> of any two portions of matter is by comparing their amounts<sub>q</sub> of motion at a single speed.

#### Proof

Matter is infinitely divisible, so the amount<sub>q</sub> of matter in a given portion can't be determined directly by how many parts it has. How much<sub>q</sub> matter there is in one portion of matter can be directly compared with how much<sub>q</sub> there is in another, if the two are of exactly the same kind, because in that case the amounts<sub>q</sub> are proportional to the volumes. But

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Proposition 1 concerns quantitative comparisons between *any* two portions of matter, including ones of different kinds. So there is no all-purpose method—direct or indirect—for comparing *any* two portions of matter with one another, if we ignore their motions. If we bring motion into the story we do get a universally valid procedure for such comparisons—and it is the only one we can have. It involves measuring the amounts of matter in terms of the amounts<sub>q</sub> of motion. But this comparison gives us what we want only if the two portions of matter are going at the same speed. Therefore etc.

#### Note

How much<sub>q</sub> motion a body has is how much<sub>q</sub> matter it has and how fast<sub>q</sub> it is moving. One body has twice the motion of another body if

- they have the same speed, and one has twice as much<sub>q</sub> matter as the other, or
- they have the same mass and one has twice the speed of the other.

That is because the determinate concept of a *size* or *amount* is possible only through the construction of the quantum; and such a construction involves putting together many items that are *equivalent* to one another [see ‘what Kant seems to have meant’ on page 14]. Thus, the construction of a motion’s amount<sub>q</sub> is the putting together of many equivalent motions. Now, in the context of phronomy there is no difference between

- giving to a movable thing a speed S, and
- giving to each of n equivalent movable things a speed of S/n.

The first thing we get from this is an *apparently* phronomic concept of the amount<sub>q</sub> of a motion, as composed of many motions that are external to one another but constitute a single united whole. And if we think of each point as getting

its •moving force from •how it is moving, this turns into a mechanical concept of the amount<sub>q</sub> of the motion. But •actually this is a blind alley; we can’t get at a mechanical concept of amount-of-motion in this way, because •in phronomy we *can’t* represent a motion as composed of many motions existing externally to one another. Why not? Because in phronomy movable items are represented as mere points, with no moving force, so that the only basis for distinguishing the amounts<sub>q</sub> of motion of two things is in terms of their •differences of speed. [In a spectacularly obscure passage, Kant goes on from there to compare measuring amounts of •motion with measuring amounts of •action, and to criticise a wrong idea that some theorists have had about the latter. His purpose in going into all this seems to be to present some thoughts about differentiating ‘dead forces’ from ‘living forces’. We hear no more of that distinction in the present work, and Kant invites us to bypass it when he ends by saying:] . . . if indeed the terminology of ‘dead force’ and ‘living force’ deserves to be retained at all.

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#### Remark

I have things to say in explanation of the preceding three statements—[i.e. Definition 2, Proposition 1, and the following Note]—and in the interests of concentration I shall condense them into a single treatment.

Definition 2 says that the quantity [*Quantität*] of a portion of matter can only be thought of in terms of how many movable parts (external to one another) it has. This is a remarkable and fundamental statement of universal mechanics, •because it supplies an answer to the important question ‘Can we have a concept of the *intensive* magnitude of an instance of moving force? The answer is that we cannot. Such an intensive magnitude would have to be independent of •the amount of matter and of •the speed, both of which

are extensive magnitudes; and Definition 2 tells us that those are the only quantitative notions that are applicable to a portion of matter. Intensive magnitude *would* have a place if matter consisted of monads. A monad has (by definition) no parts; so any monad—however it was related to anything else—could be more or less real in some way that didn't depend on how many parts-external-to-one-another it had, which means that its reality could be an *intensive* magnitude. As for the concept of mass in Definition 2: it is usually equated with the concept of quantity, but this is wrong. Fluid portions of matter can by their own motion *act in mass* [see Definition 2] but they can also *act in flow*. In the so-called water-hammer—which causes a knocking sound in the pipes when a flow of water is suddenly blocked—the water in striking acts in mass, i.e. with all its parts simultaneously; the same is true when a pot full of water is weighed on a scale. But when the water of a millstream acts on the lower paddles of the wheel, it doesn't do so in mass, i.e. with all its parts *together* colliding with the wheel; rather, the parts act successively. So if in this case we want to determine how much<sub>q</sub> matter is being moved with a certain speed and exerting moving force, we must first of all look for *the body of water*, i.e. find out how much<sub>q</sub> matter can produce the same action when it acts in mass (by bringing its weight to bear) with a certain speed. That's why we usually understand by the word 'mass' the amount<sub>q</sub> of a *solid* body (a fluid can be treated as solid on the strength of the vessel containing it). Finally, there's something odd about the Proposition and its appended Note. According to the Proposition,

•how much<sub>q</sub> matter must be estimated by how much<sub>q</sub> motion at a given speed,

whereas according to the Note,

•how much<sub>q</sub> motion...must be estimated by how

much<sub>q</sub> moved matter.

This seems to revolve in a circle, offering no prospect of a determinate concept of either of the terms. It really would be circular if these were definitions of concepts in terms of one another, but that's not what is going on. The Proposition does define a concept, but all the Note does is to explain how that concept applies to experience. . . .

This should be noted: For any given portion of matter, the question 'How much [*Quantität*] matter is there in this?' is the question 'How much [*Quantität*] **substance** is there in this?' and *not* 'How much [*Größe*] of **quality** Q is there in this?', where Q is some special quality such as the powers of repulsion or attraction that are cited in dynamics. And what is meant in this context by 'the amount [*Quantum*] of substance' is merely 'how many movable parts' there are in the given portion of matter.

[Throughout this paragraph, 'how many' translates the German noun *Menge*. Of the two other currently available English translations of this work, one says

(1) 'the mere aggregate of the movable'

while the other says

(2) 'the mere number of the movable parts'.

Of these, the second is not quite right, but is nearer to right than the other. As you might guess, *Menge* has two meanings. (1) It can be a concrete noun, meaning something like 'multitude' or 'crowd' or, if you like, 'aggregate'. 'I looked along the street and saw a *Menge* of angry people coming towards me'. (2) It also has a sense in which it is an abstract noun, meaning something like 'how-many-ness'. Why say it in that clumsy way, rather than just using 'number' as the translator did? Because Kant sometimes—notably in the *Critique of Pure Reason*—uses *Menge* as his more general how-many concept while reserving *Zahl* = 'number' to mean '*Menge* that is determinate'. He holds that when there are infinitely many Fs, the *Menge* of Fs is not determinate, and so there is no such thing as the *Zahl* of Fs; the phrase 'infinite number' is, he holds, self-contradictory. In the present work, most occurrences of *Menge* are

in the context of items of which there are infinitely or endlessly many, so that Kant couldn't have used *Zahl* (which in fact occurs only twice in the whole work). The more important point, however, is that all through this work *Menge* is being used as an abstract 'how-many' noun and not as a concrete noun meaning 'crowd' or the like.]

[What follows expands and re-arranges what Kant wrote, in ways that the usual conventions of 'small dots' can't easily indicate. The thoughts expressed here are all present, explicitly or implicitly, in the paragraph that is being replaced.] This emphasis on how-many-parts is justified by a deep theoretical point about the line between *substance* and *quality*. The concept of substance is the concept of the rock-bottom subject, i.e. a subject that isn't in its turn a quality of another subject. Now, we want to get a grip on a notion of *how much* substance there is in a given portion of matter; and we can't get at it through any such notion as that of how-much-force the portion has or how-big the portion is, because force and size belong on the quality side of the fundamental substance/quality line. Well, then, what *isn't* on that side of the line? The only candidate is how-many-ness: 'How many movable parts of substance S are there?' isn't a question about any of S's qualities, so the answer to it doesn't slide across to the wrong side of the line. [The remainder of the 'Remark' is omitted because the preparer to this version has been defeated by it. On page 67 the passage is presented in each of the two currently available English translations of it.]

### Proposition 2

*First law of mechanics:* Through all changes of corporeal Nature, the over-all amount<sub>q</sub> of matter remains the same—neither increased nor lessened.

#### Proof

(Universal metaphysics contains the proposition that through all changes of Nature no substance either comes into existence or goes out of existence; all that mechanics is adding

here is an account of what substance in matter *is*.) In every portion of matter the movable in space is the ultimate subject of all qualities that matter has, and •how many movable parts external to one another a portion of matter has is •how much substance there is to it. Hence the amount of any portion of material substance is nothing but how many substances it consists of. So the only way the amount<sub>q</sub> of matter could be increased or lessened would be for material substances to go out of existence or for new ones to come into existence. But substances *never* come into or go out of existence in changes of matter. So the over-all amount<sub>q</sub> of matter in the world is neither increased nor lessened in these changes, but remains always the same.

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#### Remark

The essential thing about substance as it figures in this proof—only as existing in space and subject to the conditions of space, and hence as having to be an object of the outer senses—is that the amount of it can't be increased or diminished unless •some• substance comes into or goes out of existence. Why not? Because if x is something that can exist only in space, the amount of x that there is has to consist in •facts about• the parts that x has external to one another, and if these are real (i.e. are movable) they must necessarily be substances. On the other hand, something regarded as an object of inner sense can as substance have an amount or magnitude that doesn't consist of parts external to one another, so that the parts that it does have are not substances. When this item comes into or goes out of existence, that doesn't involve any *substance's* doing so; so the magnitude of the item can increase or lessen without detriment to the principle of the permanence of *substance*. [On this and the next two pages Kant uses the phrase 'the permanence (*Beharrlichkeit*) of substance' to mean 'the fact that no substance comes

into, or goes out of, existence'. On pages 11–12 the cognate adjective, *beharrlich*, being used for a quite different purpose, was translated as 'time-taking'.] How can that happen? Well, I can be more conscious or less conscious, so my mental representations can be clearer or less clear, and this gives to my faculty of consciousness—I call it 'Self-awareness'—a degree of reality, and we can even say that the substance of my soul has such a degree; and none of this in any way requires that any substance come into existence or go out of existence. This faculty of Self-awareness can gradually diminish, to the point where it finally goes right out of existence, so the substance of the soul can gradually go out of existence. [In this sentence and the preceding one, Kant doesn't say that the soul is a substance; he speaks of the 'substance of the soul'. He doesn't explain the 'substance of' locution (which occurs nowhere else in this work, and nowhere in the *Critique of Pure Reason*). It does save him from having contradicted himself about whether substances can go out of existence.] If a thing has parts external to one another, the only way it could go out of existence gradually is by being slowly dismantled, pulled apart; but the soul can go out of existence gradually in a different way, through being gradually lessened and eventually extinguished. [Kant's next sentence is hard to follow. In it he sketches, in a condensed form, some doctrine from the *Critique of Pure Reason*. He is facing the challenge 'Don't we know that the soul is a substance? Isn't it obvious that when I say "I see something red" I am attributing the predicate "sees something red" to the mental thing, the substance, that I call "I"?' Kant rejects this, and gestures towards the *Critique's* account of how 'I' does work in all its uses. Fortunately, we don't really need that account for present purposes. All that matters here (and even it doesn't here matter *much*) is his negative thesis that 'I' or the German *Ich* does not serve to pick out an individual thing, and therefore isn't the name of a substance. Kant winds this

up by saying that the person who uses 'I' isn't employing any concept of himself as a substance, and he is clearly implying that there is no such concept. Then:] In contrast with that, the concept of a portion of matter as substance is the concept of *something that is movable in space*. So it's not surprising that the permanence of substance can be proved of matter but not of the soul. This is because from the concept of matter as *what is movable in space* it follows that the quantitative or *how-much* aspect of matter depends on there being many •real parts external to one another—and thus many •substances. Thus, the going out of existence of a portion of matter would involve the going out of existence of many substances, and that is impossible according to the law of permanence. [Kant has *Gesetz der Stetigkeit* = 'law of continuity' here, an obvious slip.] (The portion of matter could be diminished by being taken apart, but that isn't the same as going out of existence.) The thought 'I', on the other hand, isn't a *concept* at all but only an inner perception. And nothing *follows from* this thought (except that an object of inner sense is completely distinct from anything that is thought of merely as an object of outer sense); so the permanence of the soul as substance doesn't follow from it.

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### Proposition 3

*Second law of mechanics:* Every change in matter has an external cause. (Every motionless body remains at rest, and every moving body continues to move in the same direction at the same speed, unless an external cause compels it to change.)

#### Proof

(Universal metaphysics contains the proposition that every change has a cause. All we have to do here in mechanics is to prove with regard to matter that every change in it must

have an *external* cause.) Because matter is a mere object of outer sense, the only facts about it are facts about how portions of matter relate to one another in space; and from this it follows that the only way there can be any change in matter is through motion—i.e. through •changes from motion to rest or vice versa, or •changes in direction and speed of motion. The principle of metaphysics says that each such change must have a cause; and this cause can't be internal, because matter has no absolutely inner states or inner causal resources. Hence all change of a portion of matter is based on an *external* cause.

### Remark

544 The name 'law of inertia' should be given only to •this law of mechanics, and not to •the law that every action has an equal and opposite reaction. The latter says what matter *does*, but the former says only what it *doesn't do*, and that is a better fit for the word 'inertia'. To say that matter 'has inertia' is just to say that matter in itself is *lifeless*. For a substance to have *life* is for it to be able to get itself, through its own inner resources, to *act*—i.e. to change in some way (for any finite substance) or start or stop moving (for any material substance). Now, the only inner resource we know of through which a substance might change its state is *desire*, along with its dependents—•feelings of pleasure and unpleasure, •appetite, and •will—and the only inner activity that we know of is *thought*. But none of these causes and activities have anything to do with the representations of outer sense, and so they don't belong to matter as matter. Therefore all matter as such is lifeless; and that is what Proposition 3, the one about inertia, says—and it's *all* it says. If we want to explain any change in a material thing in terms of *life*, we'll have to look for this cause in some other substance that is different from matter although bound up

with it. That's because in gathering knowledge about Nature we must •first discover the laws of matter as such, not mixing them up with any other active causes, and •then connect these laws with any other causes there may be, in order to get a clear view of exactly what each law of matter brings about unaided. The possibility of a natural science proper rests entirely on the law of inertia (along with the law of the permanence of substance). Hylozoism [= 'the thesis that matter itself is alive'] is the opposite of this, and is therefore the death of all natural philosophy! Just from the concept of inertia as •lifelessness we can infer that 'inertia' doesn't signify a thing's •positive effort to maintain its state. Only living things can be called 'inert' in this positive sense; it involves their having a thought of another state that they don't want to be in and do their best to avoid.

### Proposition 4

*Third mechanical law:* In all communication of motion, action and reaction are always equal to one another.

### Proof

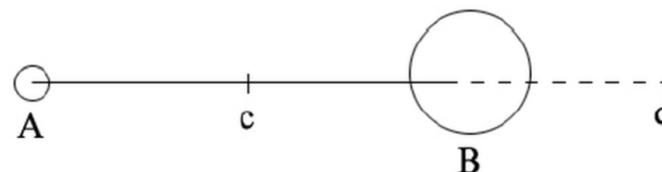
[In a notably obscure explanation—omitted here—of why he had to deal with this third law, 'for the sake of completeness', Kant refers to it as 'the law of two-way causal interaction of universal metaphysics'. His word for 'two-way causal interaction' is *Gemeinschaft*, which is standardly but unhelpfully translated as 'community'.] Active relations of portions of matter in space, and changes of these relations, have to be represented as reciprocal if they are to be •thought of as •causes of certain effects. Now, any change of such relations is *motion*; so we get the result that whenever one body causes a change in another body, the other must also be in motion (so that the interaction can go both ways); so we can't allow for any case in which one body A causes motion in another body B which until that moment was •absolutely

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at rest. What we can do is to represent B as being at rest •relative to the space to which it is referred; B must be represented as moving with its reference-space towards A, moving at the same speed in absolute space as A is moving towards B. For the change of relation (and hence the motion) is completely reciprocal between both bodies; by as much as A approaches every part of B, by that much B approaches every part of A. What we are dealing with here is not the empirical space surrounding the two bodies but only the line stretching between them (because our whole topic is just the effect that the movement of each has on the state of the other, and for that we can abstract from all relation to empirical space); and therefore we think about their motion only in terms of absolute space, in which they share equally in the motion attributed to A, the one in relative space, because there's no basis for attributing more motion to A than to B. On this footing, the motion of a body A toward an immobile body B is handled in terms of absolute space, i.e. the motion in question is treated as a relation of two causes interacting with one another and not with anything else; and so the motion which appears to us as only A's is considered as something shared between A and B. This can occur only in the following way. The speed which in the relative space is attributed only to A is divided between A and B in inverse proportion to their masses; A's share is only its speed in absolute space, whereas B (*along with the relative space in which it is at rest*) is assigned its speed in the opposite direction; and in this way the same appearance—i.e. the appearance that A moves towards B, which is motionless—is perfectly retained. [We are about to see Kant representing speeds by lines, in accordance with his statement on page 11 that 'In phoronomy we use the word "speed" with a merely spatial meaning—the measure of how far a thing travels in a given period of time'—which has the result: the longer the line, the faster the motion.] What happens in the

two-way causal interaction of the bodies is constructed as follows.

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Let a body A be moving into a collision with the body B with a speed = AB with regard to the relative space in relation to which the body B is at rest. Let the speed AB be divided into two parts, Ac and Bc, in such a way that their respective speeds are inversely proportional to their respective masses. Represent A as moved with the speed Ac in absolute space, and the larger body B (*together with the relative space*) as moving with the smaller speed Bc in the opposite direction. Thus the two motions are opposite and equal to one another. [Kant is relying here on the thesis (page 49) that in mechanics the concept of *how much motion* is a function of speed and mass.] Because they are equal and opposite, neither is the winner, and they destroy one another and both come to be, relatively to one another, i.e. in absolute space, in a state of rest. [In a helpful footnote in his translation of this work (Cambridge University Press 2004), Michael Friedman points out that Kant is here discussing the collision of perfectly inelastic bodies, i.e. ones that have no bounce-back from a collision.] So we have B moving *with its relative space* in the BA direction, and losing its motion when it collides with A; but the collision doesn't automatically cancel the motion of B's relative space as well. So we have two equivalent ways of describing the state of affairs after the collision:

- The bodies A and B are now at rest in absolute space, and relative to them the relative space moves in the direction BA with the speed Bc.

•The bodies A and B move with equal speed  $Bd = Bc$  in the direction AB, i.e. the direction that the impacting body A had.

Now, according to this,

the amount<sub>q</sub> of motion of B in the direction and with the speed Bc,

which is the same as

the amount<sub>q</sub> of motion of B in the direction Bd with the speed Bd,

is equal to

the amount<sub>q</sub> of motion of the body A with the speed and in the direction Ac.

Consequently, the effect of the collision, i.e. the motion Bd that B receives by impact in relative space, and hence also the action of the body A with the speed Ac, is always equal to the reaction Bc. That is a part of what was to be proved—the part saying that whenever a moving body causes a change in the motion of a stationary body, action and reaction are equal. It's a thesis in mathematical mechanics that this same law about action and reaction holds for the impact of one moving body on another *moving* body just as well as it does for the impact of one body on another that is *motionless*. Also, the communicating of motion by *impact* differs from the communicating of motion by *traction* only in the direction in which the portions of matter oppose one another in their motions. From all of this it follows that in all communication of motion, action and reaction are always equal to one another: an impact can communicate the motion of one body to another only by means of an equal counter-impact, a pressure only by means of an equal counter-pressure, and a traction only by an equal counter-traction).<sup>3</sup>

<sup>3</sup> In **phoronomy** a body's motion was considered merely as the change in its relation to space, so we could say that 'the body moves in *this*

#### Note 1

From this we can infer a natural law that is of some importance for universal mechanics, namely that *every* body, however great its mass may be, must be movable by the impact of *any* other body, however small its mass or speed may be. This is because the motion of body A in the direction AB *must* encounter an equal motion of B in the opposite direction BA [this refers to the diagram on page 54]. The two motions cancel one another in absolute space by impact. But thereby both bodies receive a speed  $Bd = Bc$  in the direction of the impacting one; consequently, the body B is

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direction' or 'the body stays still and the space moves in *that* direction', i.e. the opposite direction; these two were indistinguishable by any appearance. The quantity of motion of the space was merely its speed, so the quantity of motion of the body was nothing but *its* speed (which is why we could regard the body as a mere movable point). But in **mechanics** we deal with a body x that is moving relative to another body y and, through that motion, is *causally* related to y. Whether by •moving towards y and exercising its force of impenetrability or by •moving away from y and exercising its force of attraction, x comes to be in a two-way causal interaction with y. So here in mechanics there is a difference between 'x moves this way' and 'x is stationary while the space containing it moves the opposite way'. That is because we are now working with a different concept of *quantity of motion*—it involves not merely a thing's speed but also the thing's quantity of substance, which is relevant to its role as a moving cause. And we now •have to assume that both bodies are moved (in phoronomy we had our •choice about that), and indeed that they are moved with the same quantity of motion in opposite directions. When one body is not moving in relation to its space, we have to attribute the required motion to this body together with its space. For the only way the body x can act on body y other through x's own motion is by repulsive force as it approaches y or by attractive force as it moves away. Now, given that the two forces always act with equal strength and in opposite directions, any action by one of those forces in one body requires a counter-action by the other; so no body can pass motion on to an absolutely immobile body; the second body *must* be moved in the opposite direction . . .

movable by every force of impact, however small.

**Note 2**

This is, then, the **mechanical** law of the equality of action and reaction. It is based on the fact that motion is never communicated ·from one body to another· except in a two-way causal interaction between the two. And on the following: It is trivially obvious that a body A can't hit a second body B that is *motionless in relation to A*; what happens is that A hits B which is *motionless in relation to B's space*. So when A hits B, it must be that B *together with its space* is moving towards A. How fast? Well, the initial speed that we would attribute to A if we thought of it as moving in absolute space has to be divided between A and B-with-its-space, ·so that we get the right account of how long it takes A to reach B from the given distance away·; and the division must assign to each body not the same speed but the same amount of motion, so that the speed of each is inversely proportional to its mass. [See the discussion of amount-of-motion on page 49]. . . .

There is also a **dynamical** law of the equality of the action and reaction of portions of matter. It doesn't concern one portion A's sharing its motion with another portion B, but rather A's giving its entire motion to B and having motion produced in itself through B's resistance. This can be easily demonstrated in a similar way. For if A attracts B, then A compels B to approach A, i.e. resists the force with which B tries to pull away. But there's no difference between •B's pulling away from A and •A's pulling away from B; so traction and countertraction are equal to one another. 549 Similarly, if A repels B, then A resists the approach of B; but B's approaching A is just the same as A's approaching B, so it is just as correct to say that B equally resists the approach of A; so pressure and counterpressure are always equal.

**Remark 1**

This then is the construction of the communication of motion. This construction necessarily carries with it the law of the equality of action and reaction. Newton didn't venture to prove this law *a priori*, but appealed to experience to prove it. Others tried to secure this law by introducing into natural science a special force of 'inertia' (Kepler's name for it); so basically they were also deriving it—i.e. the law of action and reaction—from experience. Yet others tried to get it from the mere concept of *the communication of motion* [see the note on page 38]. They thought of this as involving a gradual transfer of body A's motion into another body B, so that A loses exactly as much as B gains; ·their view was that· the transfer stops when A and B are moving at the same speed in the same direction as that of the latter; and that rules out any reaction, i.e. any reacting force of B acting back against A that collides with it.<sup>4</sup> ·That is bad enough, but there is

<sup>4</sup> ·This theory didn't have to say that the motion-transfer is gradual·. The equality of A's action with B's . . . is secured just as well if the transfer of motion is supposed to be instantaneous, with body A coming to rest immediately after the collision; and that's the form these theorists' account would *have to take* if they were thinking of the two bodies not as elastic but as absolutely hard. But the law of motion that *that left them with* doesn't square with experience and isn't even consistent, so their only way out would be to deny that there are any absolutely hard bodies (thus making their law *contingent*, because dependent on a special quality of colliding bodies). But I can't see how the transfer-of-motion theorists could explain what happens in collisions if the colliding bodies are elastic. It is clear that when elastic body A collides with immobile elastic body B, it is *not* the case that B merely receives motion that A loses; rather, B exercises actual force in the opposite direction against A, as though it were pushing against a spring lying between them; and for this it requires just as much actual motion (but in the opposite direction) as A needs for its part in this transaction. In my version of this law, on the other hand, ·no such difficulty arises, because· it doesn't make the slightest difference whether the colliding bodies are absolutely hard

550 worse. They don't show what their account *means*, i.e. they haven't explained 'communication of motion' in a way that shows that such communication is possible. [In the original, this 'Remark' down to here is a single sentence.] The mere words 'transfer of motion from one body to another' don't explain anything. If they are understood literally, implying that motion is poured from one body into another like pouring water from one glass into another, they conflict with the principle that *qualities don't wander from one substance to another* [Kant gives this in Latin]. If that literal reading is rejected, then the theorists I am discussing must face the problem of how to make this possibility conceivable. . . . The only way to make sense of A's motion's being necessarily connected with B's motion is by attributing to both bodies dynamic forces (e.g. the force of repulsion) that precede all motion—i.e. so that the forces explain the motions, not vice versa. And *then* we can prove that the motion of A towards B is necessarily connected with the approach of B toward A; and if B is regarded as immobile, then A's motion is connected with the motion of B-together-with-its-space towards A, so that the bodies with their (basic) moving forces are considered to be moving relatively to one another. We can fully grasp this course of events *a priori*: whether or not the body B is moving in relation to its empirically knowable space, we have to regard it as moving in relation to the body A, and indeed moving in the opposite direction. Otherwise, A's movement couldn't bring into action the repelling forces of itself and of B, in which case portions of matter couldn't act mechanically on one another in any way, i.e. there couldn't

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or not. [Kant is referring to his version of the 'third mechanical law'—Proposition 4 on page 53. So his topic in this footnote, where he writes about 'the law' that these transfer-of-motion theorists have to accept, is really their *version of*, their *presentation of*, the third mechanical law.]

be any communication of motion through collisions.

### Remark 2

The name 'force of inertia' must be dismissed from natural science (despite the fame of its inventor). This must be done •because the phrase is inherently self-contradictory; and because because the •so-called• 'law of *inertia*' (law of lifelessness!) could easily be confused with the law of *reaction*; and above all •because this confusion would support and encourage the wrong account given by those who don't have a proper grasp of the mechanical laws. According to *their* account, the reactions of bodies—now described as •exercises of• 'the force of inertia'—would lead to

(i) the lessening or annihilation of all the motion in the world,

and to

(ii) collisions in which no motion is communicated.

The reason for (i) is this: according to the account that I am attacking, the moving body A would have to 'spend' some of its motion in overcoming the inertia of the immobile body B, and that 'expense' would be sheer loss. And the reason for (ii) is this: If B were very massive •and A much less so•, A wouldn't have enough motion *both* to overcome B's 'inertia' *and* then to make B move; so that this would be a collision in which no motion was communicated. •Summing up: 551

A motion can't be resisted by anything except an opposite motion; it can't be resisted by a body's immobility! So the 'inertia' of matter, i.e. its mere incapacity to get itself moving, isn't the cause of any resistance. It could be defined:

'inertia' = 'a unique force to resist a body but not to move it'

and that would make 'inertia', •despite its definition•, a word without any meaning. We could put 'inertia' to a better use by designating the three laws of universal mechanics as:

- the law of matter's subsistence [Proposition 2, page 51],
- the law of matter's inertia [Proposition 3, page 52], and
- the law of the reaction of portions of matter [Proposition 4, page 53].

These laws, and hence all the propositions of mechanical science, correspond exactly to the categories of •substance, •causality, and •two-way interaction. There is no need for me to discuss this here.

### GENERAL REMARK ON MECHANICS

The communication of motion takes place only by means of moving forces—impenetrability and attraction—that a portion of matter also has when it is not moving. The action of a moving force on a body at an instant is the solicitation of the body. [That's the first appearance of 'solicitation' in this work. It is or was a technical term in mechanics. You can safely think of it as meaning 'instantaneous tug or push'.] The speed of the body brought about by its solicitation—understood in terms of how this speed can increase uniformly through time—is the acceleration-at-a-moment value. (The latter must involve only an infinitely small speed, because otherwise the body would attain through the acceleration value an infinite speed in a given time, which is impossible. . . .) As an example of the solicitation of matter by expansive force, let us consider compressed air holding up a weight. In this situation, the air's exercise of expansive force must have a finite speed. [In this paragraph, 'finite' always means 'more than infinitely small'.] Why? Because expansive force occurs only at the surface, which means that it is the motion of an infinitely small amount of matter; and so we have on the air's side of the transaction

- an infinitely small amount of matter with a finite speed,

which has to balance, or to equal, what there is on the weight's side, namely

- a body of finite mass with an infinitely small speed.

Whereas expansive force operates only at the surface, and is therefore a force exercised by an infinitely small amount of matter, attraction is a penetrating force: a body's attractive force penetrates the body itself, so that the body's inner parts contribute to the attractive force of the body as a whole. If attraction were not a penetrating force, the equations implied by the mechanical proposition 4 wouldn't come out right. [That somewhat simplifies what Kant wrote.] *Cohesion* is often thought of as a force operating only at surfaces; but we now see that if cohesion is to be true attraction and not merely external compression—i.e. if it's to be thought of in terms of the parts of a body *pulling* together rather than being *pushed* together—it can't be thought of in this way.

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An absolutely hard body would be one whose parts attracted one another so strongly that no weight could •separate or •re-arrange them. This means that the parts of such a body would have to pull on one another infinitely more strongly than gravity pulls on them (•because: *however* strong the gravitational pull, the part-on-part pull will defeat it). But . . . [and then Kant proceeds with a defeatingly technical reason why this fact, conjoined with some others, implies that absolute hardness is impossible. He follows this with what seems to be an entirely different and much more accessible reason, namely:] An absolutely hard body is impossible because

in a collision between body x and absolutely hard body y, x would be moving with a finite speed and y would react *instantaneously* with a resistance equal to the whole of x's force.

And this is impossible. A portion of matter produces by its impenetrability or cohesion only an infinitely small instan-

taneous resistance to the force of a body that collides with it. A consequence of this is the mechanical law of continuity, namely:

A collision can't make a body move, or stop moving, or change speed or direction, *instantaneously*. Any such change occurs through a time-taking infinite series of intermediate states whose difference from one another is smaller than that between the first and last such states.

[That is what Kant wrote; but the reference to 'the first and last' states doesn't help to pin down the notion of continuity, and may be a mere slip on Kant's part. What he needed, and perhaps what he meant, was this: 'Given *any* two members of this series, there is an intermediate state that is more like each of them than they are like one another.']

Thus, a moving body *x* that collides with a portion of matter *y* is halted by *y*'s resistance not *instantaneously* but through a continuous slowing down; similarly for the other changes that a collision can subject a body to—starting to move, changing speed, changing direction. When the direction of a body's motion changes, it goes through all possible directions intermediate between its first and last ones, which means that it changes direction by moving in a curved line. This law of continuity also applies. . . .to changes in a body's state by means of attraction. This mechanical law of continuity is based on the law of the inertia of matter. On the other hand, the *metaphysical* law of continuity applies quite generally to all change (internal as well as external), and its basis is provided by concepts: the concept of *change* as a magnitude, and the concept of *generation of a magnitude* (which necessarily happens continuously through a period of time). So this metaphysical law has no place here.

## Chapter 4 Metaphysical Foundations of Phenomenology

### Definition

**Matter** is whatever is movable and can be an object of experience. 554

### Remark

Like everything that is represented through the senses, motion is given only as appearance. For the representation of motion to become experience, there has to be—in addition—to (i) what is received through the senses—also (ii) something thought by the understanding. As well as (i) being a state of the perceiving subject, the representation of motion must also determine an object. So something movable becomes an object of experience when a certain object (here, a material thing) is thought of as falling under the predicate 'moves'. Now, motion is change of relation in space. So there are always two correlates here, namely matter and space, and we have some options:

- (a) In appearance, we can handle things in terms of the motion of matter or the motion of space; it doesn't matter which we choose, because the two accounts are equivalent.
- (b) In the experience of motion we must think of one of the two—matter or space—as moving and the other as staying still.
- (c) Reason must necessarily represent both of these correlates as moving at the same time.

All we get in the appearance of motion is the change in the relation of matter to space; and that doesn't pick out any of those three options as the right one. But we can't 555

just leave it at that: we have to settle the conditions under which a movable thing can be thought of as moving in this or that specific way, because without that there can't be *experience* of a moving thing. (The difference that I am invoking here between •appearance and •experience is not the same as the difference between •illusion and •truth—i.e. the difference between •how things seem and •how they are. That's because •illusion or seeming is nothing like •appearance: something's seeming to be the case always involves judgments about what is objectively the case; such judgments are always in danger of going wrong by taking the subjective to be objective, but in appearance there is no judgment of the understanding. This distinction is significant not only here but all through philosophy, because there is always confusion when what is said about 'appearances' is taken to be referring to illusion or seeming.)

### Proposition 1

(a) 'That portion of matter is moving in a straight line in relation to that empirical space'—as distinct from 'The space is moving in the opposite direction in relation to the portion of matter'—is a merely possible predicate. (b) 'That portion of matter is moving in a straight line *period*', i.e. its movement is absolute, not thought of as a changing relation to matter outside it, is impossible. [Kant is assuming here that if you don't relate a moving thing to any body outside it you can't be relating it to an empirical space.]

#### Proof

In the case of a body *x* moving in a relative space *y*, these two:

- (1) *y* is at rest and *x* is moving *this* way within it, and
- (2) *x* is at rest and *y* is moving in *that* way—the opposite way—around it

tell the same story about what is objectively happening out there; they differ only in what they imply about the subject, the person whose experience is being reported on. So there is no difference between them at the level of *experience*, only at the level of *appearance*. If the spectator puts himself into the space *y*, then he says that the body moves; if he puts himself (at least in thought) into another space *z* that encloses *y*, with *x* being at rest in relation to *z*, then he (the spectator) will say that space *y* is moving. Therefore, in experience... there is no difference whatever between (1) and (2). [In a very repetitious passage, Kant belabours the point that (1) and (2) represent a pair of *choices* that one might make, not rival accounts of what is objectively the case. And yet they do apply conflicting predicates to *x*—'moving' and 'at rest'—from which Kant concludes:] Something that is in itself undetermined as regards two mutually opposed predicates is to that extent merely possible. So the straight-line motion of a portion of matter in empirical space—as against the opposite motion of the space—is in experience a merely possible predicate. This was (a) the first thing to be proved.

Next: For any •relation to be an object of experience, each of the related items must be an object of experience; this holds also for any •change of relation, including the special case of the relation-change that is •motion. Now pure space (in contrast to empirical space), i.e. absolute space (in contrast to relative space) is not an object of experience; basically it is nothing. So straight-line motion without reference to anything empirical, i.e. absolute motion, is utterly impossible. This was (b) the second thing to be proved. . . .

#### Remark

This proposition determines the modality of motion with

respect to *phoronomy*—namely *possibility*.

### Proposition 2

557 The circular motion of a portion of matter x, as against the opposite motion of the space, is an actual predicate of x. On the other hand, the opposite motion of a relative space, taken as a substitute for the motion of x, is not an actual motion of x—at most it may *seem* to be an actual motion of that body, but this is a mere illusion.

#### Proof

Circular motion is (like every curved-line motion) a continuous change of •straight-line motion; and since •this motion is itself a continuous change of relation to external space; so circular motion is a *change of the change* of these external spatial relations, and is therefore a continuous arising of new motions. Now, according to the law of inertia, a motion can't start up without having an external cause. But the circulating body x at every point of this circle is (also by the law of inertia) endeavouring to proceed in the straight line at a tangent to the circle, and this straight-line motion acts *against* the external cause •of x's circular movement•. [Re 'endeavouring', see the long note on page 19.] Hence every body in circular motion manifests by its motion a moving force. Now, the motion of the space, in contrast to the motion of the body, is merely phoronomic and has no moving force. Consequently, the judgment that here either the body is moved or else the space is moved in the opposite direction is a disjunctive one, by which, if the one member, namely, the motion of the body, is posited, then the other member, namely, the motion of the space, is excluded. Therefore, the circular motion of a body, in contradistinction to the motion of the space, is an actual motion. Even though according to the appearance [*Erscheinung*] •the motion of the space agrees

with •the circular motion of the body, nevertheless in the complex of all appearances, i.e. of possible experience, the •former motion conflicts with the •latter; and hence the former is nothing but mere illusion [*Schein*].

#### Remark

This proposition determines the modality of motion with regard to dynamics—namely, *actuality*. For a motion that can't take place without the influence of a continuously acting external moving force exhibits—directly or indirectly—basic moving forces of matter, either of repulsion or of attraction. In connection with this topic, see Newton's scholium to the definitions at the start of his *Mathematical Principles of Natural Philosophy*. This makes it very clear that

the circular motion of rotating around a common 558  
centre,

and therefore also

the rotation of the earth on its axis,

can be known by experience even in empty space; which means that a motion that is a change of external relations in space can be empirically given, even though this space itself is not empirically given and is not an object of experience. This paradox deserves to be solved.

### Proposition 3

In every motion of a body whereby it is moving with regard to another body, an opposite and equal motion of this other body is necessary.

#### Proof

According to the third law of mechanics (Proposition 4 on page 53), the communication of the motion of the bodies is possible only through the two-way causal interaction of their basic moving forces, and this two-way causal interaction is possible only through mutually opposite and equal motion.

So the motion of both bodies is actual. But the actuality of this motion doesn't come from. . . .the influence of external forces, but follows immediately and inevitably from a *concept*—the concept of *how something that moves relates to each other thing that can be moved by it*. So motion of the 'other thing' is necessary.

### Remark

This Proposition determines the modality of motion with regard to mechanics, namely, *necessity*. It is immediately obvious that these three propositions determine the motion of matter with regard to its possibility, actuality, and necessity, and thus hence with regard to all three categories of modality. [This completes Kant's attempt to tie his four chapters severally to his four trios of categories.]

### GENERAL REMARK ON PHENOMENOLOGY

So we have here three concepts that have to be employed in universal natural science, and which therefore have to be understood in precise detail—though the details are hard to pin down and hard to understand.

They are these:

- (1) the concept of motion in relative (movable) space;
- (2) the concept of motion in absolute (immovable) space;
- 559 (3) the concept of the across-the-board distinction between relative motion and absolute motion.

The concept of absolute space lies at the foundation of all of these. How do we come by this unusual concept, and why do we *have to* use it?

•The concept of absolute space can't be a concept of the **understanding**, because absolute space can't be an object of **experience**—space without matter isn't an object of perception. But •it is a necessary concept of **reason**, and that gives it the status of an *idea*, but that is all it is—a mere

idea. [See the note on 'idea' on page 9.] •Here is how it comes into play•. For there to be even an *appearance* of motion, there has to be an empirical representation of the space with which the moving thing is changing its relation; but *that* space—the space that is perceived—must be *material* and therefore itself movable. [Kant says that this last 'therefore' depends on 'the concept of matter in general'. Perhaps he is referring to the equation of 'material space' with 'relative space' in the phoronomic Definition of 'matter' on page 7.] Now, we can't think of this space as moving except by thinking of it as contained in a more extensive space that is at rest. But this latter space can be related in just the same way to a still larger space . . . and so on to infinity, without ever arriving empirically at an immovable (immaterial) space with regard to which any portion of matter could be said to be outright moving or at rest. Rather, we have to keep changing our concept of these relational set-ups depending on what we are thinking of as moving relative to what. •I'll say it again•:

The condition for regarding something as at rest, or as moving, is always its being placed in a relative space—*always*, again and again ad infinitum, as we enlarge our view.

From this we can draw two conclusions: **(1)** All motion or rest must be merely relative; neither can be absolute. That is, matter can be thought of as moving or at rest only in relation to •matter and never in relation to •mere space without matter. It follows that absolute motion—i.e. motion that doesn't consist in one portion of matter changing its relation to another portion—is simply impossible. **(2)** For this very reason, there can't be, out of all the ever-wider concepts of motion or rest in relative space, one that is •so wide as to be• valid for every appearance. •To have such an all-purpose concept•, we have to make room in our minds for the thought of a space that isn't nested within any larger space, i.e. an

560 **absolute** space in which all relative motions are nested. In such a space everything empirical is movable, . . .<sup>5</sup> but none can be valid as absolute motion or rest. . . . So absolute space is necessary not as a **concept** of an actual object but as an **idea** that is to regulate all our thoughts about relative motion. If we want all the appearances of motion and rest to be held together by a determinate empirical concept, we must put them within the framework of the idea of absolute space. [Actually, Kant writes that these appearances must all *auf den absoluten Raum reducirt werden* which literally = ‘be reduced to absolute space’; but his meaning, in this sentence and the next, seems to be something about framing or handling-in-terms-of.]

Thus the straight-line motion of a body in relative space is handled in terms of [*reducirt auf*] absolute space **(i)** when I think of a body as being at rest and think of the relative space ·that it is in· as moving in the opposite direction—moving in non-empirical absolute space—and **(ii)** when I think of the body as moving and the relative space as being motionless in absolute space·. The two ways of representing the situation are empirically exactly alike. By means of this representation all possible appearances of rectilinear motions which a body

<sup>5</sup> [Kant offers here a longish footnote, to the following effect: Any empirically knowable fact about something’s moving can be construed either as a body moving in a relative space or a relative space moving around a body. In the context of phoronomy, these two are alternatives; we shouldn’t think of them as a disjunction—‘P or Q, one of which must be wrong’—because the difference between them is a difference of viewpoint, a difference in how the knowing *subject* relates to the state of affairs, not in what is *objectively* the case. In the context of dynamics, on the other hand, such a pair of propositions are rivals, which can’t both be true. And in mechanics there is a different pattern again: When one body is rushing towards another, we must attribute an equal proportion of the total motion to each body. Kant is here presenting a trio of ways of understanding a certain thesis of the form ‘P or Q’: disjunctively (dynamics), distributively (mechanics), alternatively (phenomenology).]

might simultaneously have are grounded in the concept of experience that unites them all, namely the concept of merely relative motion and rest.

According to Proposition 2 [page 61], circular motion can be experienced as actual motion, even if no external empirically given space comes into the story; so it seems to be absolute motion. ·I’ll say that again, explaining it a little as I go·. A motion such as **(a)** the earth’s rotation on its axis relative to the stars is an appearance that can be matched by **(b)** the opposite motion of the space of the stars, and these two are ·empirically· fully equivalent. But Proposition 2 forbids us ever to postulate **(b)** instead of **(a)**; so **(a)** is not to be represented as externally relative—which sounds as though it is being assumed to be absolute.

But ·that’s a mistake·. What we are dealing with here is the ·humdrum everyday· difference between what seems to be the case and what is really the case, not the ·metaphysical· distinction between relative space and absolute space. Employing the former distinction, we can and do have empirical evidence that the earth is really spinning and thus that the stars may be at rest, although the space they move in can’t be perceived. The earth’s circular motion doesn’t present us with any appearance of change of place, i.e. any ·phoronomic change in the earth’s relation to the (empirical) space surrounding it; but it exhibits a continuous ·dynamic change in the relations amongst portions of matter **within the space that it occupies**, and this change is provable by experience. For example, the attraction ·that holds the earth together· is constantly lessened by an endeavour to escape, ·i.e. by centrifugal force·; we know about this empirically, and it’s a result of the earth’s rotation, which shows that the rotation is real and not illusory. [Kant wrote this paragraph down to here as a single sentence.] Thus, for instance, we can represent the earth as spinning on its axis in infinite empty

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space, and can produce empirical evidence for this motion even though it doesn't involve any phoronomic change (i.e. change in the appearance) in how the earth's parts relate to one another or to the space surrounding the earth. . . . Here is a description of one course of events that would provide such evidence. I let a stone fall down a deep hole running to the centre of the earth; and I find that although gravity keeps taking it downwards, its fall continuously diverges from the vertical direction by tending towards the east; from which I conclude that the earth is rotated on its axis from evening to morning. . . . This is good enough evidence of the earth's actually rotating in that way; and we don't get such evidence from the change in the earth's relation to external space (the starry heavens). Why not? Because that change is a mere appearance, which could come from either of two opposed causes—from the earth's spinning on its axis or from the stars revolving around the earth. . . . But the earth's rotation, even though it isn't a change of relation to empirical space (I am now returning to the imagined case of a rotating world in a space that is otherwise empty), isn't a case of absolute motion. Rather, it is a continuous change in how portions of matter relate to one another, so it really is only a case of relative motion, although we represent it to ourselves as as happening in absolute space. And it's just because this movement of the earth is relative that it is true or actual motion.

[Kant is here recalling us to his point that the line between

**(a1)** illusory and **(a2)** real or actual

is not the same as the line between

**(b1)** how things appear and **(b2)** how they are in themselves, or the line between

**(c1)** relative space and **(c2)** absolute space.

He is emphasizing the difference by saying that the status of the earth's rotation as something **(a2)** real depends on its belonging to **(b1)** the realm of appearance.] Our evidence that this rotation is true or

actual rests upon our encounter with the fact that parts of the earth outside its axis of rotation tend to fly off, i.e. the fact that any two parts of the earth that are exact antipodes of one another tend to move apart. . . . [Kant likens this to a slightly different consideration that Newton used—two bodies joined by a cord and rotating, pulling on the cord—see his remark about a 'paradox' on page 61.] 562

As for the third proposition: to show the truth or actuality of the motions of two bodies moving relatively to one another, showing this without reference to empirical space, we don't need to learn from experience about an active dynamical influence (of gravity or of a taut cord), though we needed this in the case of the second proposition. Rather, we can get this result from the mere dynamical possibility of such an influence, as a property of matter (repulsion or attraction). That possibility brings with it the result that any motion by one of the two bodies is matched by an equal and opposite motion of the other at the same time. indeed such action and reaction stem from mere concepts of a relative motion when this motion is regarded as in absolute space, i.e. according to truth. Therefore, this third proposition is, like everything adequately provable from mere concepts, a law of an absolutely necessary countermotion.

So there is no absolute motion even if a body in absolute space is thought of as moving in relation to another body. The motions of the two bodies are here not relative to the space surrounding them but only to the space between them, which is the sole determinant of their external relation to one another. . . . So these motions are only relative. Thus, absolute motion would have to be motion that a body has without a relation to any other matter, and the only candidate for this role would be the straight-line motion of the universe, i.e. of the system of all matter. It is easy to see why: If outside of a portion of matter x there is any other matter,

even if separated from *x* by empty space, then *x*'s motion would certainly be relative. Thus, if you can show regarding any law of motion *L* that

denying *L* implies that there is a straight-line motion of the whole universe,

563 that proves that *L* is absolutely necessary, because such motion is utterly impossible. There is a law of that kind, namely the law of the reaction of portions of matter in all two-way causal interactions that depend on motion [see Proposition 4, page 53]. Any divergence from this law would consist in a shove in one direction without an equal shove in the opposite direction; so it would create a straight-line movement of the common centre of gravity of all matter, and hence of the whole universe. No such result follows from the thesis that the entire universe rotates on its axis; so there is never any obstacle to thinking of the universe in this way, though I can't see any conceivable use for it.

Corresponding to the three concepts of motion and moving forces, there are three concepts of empty space. **(1)** What passes for 'empty space' (or 'absolute space') in the context of **phoronomy** really shouldn't be called empty space. It is only the *idea* [see the note on 'idea' on page 9] of a space from which I filter out all particular matter that would make it an object of experience, in order to think of it as the space within which every material or empirical space can move; this being something I want so as to think of every truth of the form '*x* moves' not as predicating something of *x* alone but as relating *x* to something else. So this 'ideal' space belongs not to the existence of things but merely to the fixing of concepts; so no empty space on this pattern exists. **(2)** in the context of **dynamics**, empty space is space that isn't filled, i.e. space in which things move without being resisted by other things, i.e. a space in which no repelling force acts. Such a space might be either empty space within

the world or outside of the world (if the world is represented as limited). An empty space within the world can be further subdivided into

- (a)** scattered all through the world, so that a part of the volume of any body may be empty space; and
- (b)** occurring between bodies, e.g. as space between the stars.

This distinction is not theoretically deep, because it doesn't mark off different *kinds* of empty space but only different places in the world where empty space might occur. Still, the distinction is put to use, because the two sides of it are used for different explanatory purposes. **(a)** Space within bodies is used to explain differences in the density of bodies; and **(b)** space between bodies is used to explain how motion is possible. It isn't necessary to **(a)** assume empty space for the first purpose, as I have shown in the General Remark on Dynamics [see pages 39–41 and 46–47]; but there's no way of showing that empty space is impossible because its concept is self-contradictory. Still, even if it can't be ruled out on merely logical grounds, there might be general physical grounds for banishing empty space from the doctrine of Nature. . . . Suppose that the following turned out to be the case (there are many reasons for thinking that it is the case):

What holds bodies together is not true but only apparent attraction; what really holds a body together is pressure from the outside, pressure from matter (the ether) that is distributed everywhere in the universe. What leads this matter to exert this pressure is *gravitation*, this being a basic attraction that all matter exerts.

If this is how things stand, then empty space within portions of matter would be impossible—not logically but dynamically impossible, and therefore physically impossible. Why? Because in this state of affairs every portion of matter would

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expand into the empty spaces assumed to be within it (because there's nothing here to resist such expansion), so that those spaces would always be kept filled up. As for **(b)** an empty space outside of the world (i.e. outside the totality of . . . large heavenly bodies) would be impossible for the very same reasons. In the scenario we are exploring, these large bodies are surrounded by ether which, driven by the attractive force, presses in on the stars and maintains them in their density. The further any portion of this ether is from the star-totality that we are calling 'the world', the less dense it is; this lessening of density continues ad infinitum as the distance grows; but it never gets to the point where the density is zero and that portion of space is therefore empty. [Kant does not try to explain why the density of portions of ether is proportional to their distance from 'the world'.] Don't be surprised that this elimination of empty space is in the meantime entirely hypothetical; the assertion that there is empty space doesn't fare any better! Those who venture to decide this controversial question dogmatically, whether for empty space or against it, basically rely on nothing but metaphysical suppositions, as you'll have noticed in the dynamics; and I had at least to show here that the question can't be answered by metaphysics. **(3)** Concerning empty space in a **mechanical** sense—i.e. the supposed emptiness accumulated in the universe to provide the heavenly bodies with room to move—it is obvious that the possibility or impossibility of this doesn't rest on metaphysical grounds but on Nature's secrets (so hard to unravel!) concerning how matter sets limits to its own force of extension. . . .

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This brings us to the end of the metaphysical doctrine of body, and we end with the *empty* and therefore with the *inconceivable*! On this topic, the doctrine of body meets the

same fate as every other attempt by reason to get back to the principles of the first causes of things. It fails in these attempts because it brings to them its own nature, which is such that the only things it can grasp are ones that are specified as satisfying certain conditions, and yet it can never be satisfied with anything conditioned. When it is gripped by a thirst for knowledge that invites it to reach for the absolute totality of all conditions, all it can do is to turn back from *objects* to *itself* in order to investigate and determine the ultimate boundary of its powers, instead of investigating and determining the ultimate boundary of things.

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THE PASSAGE THAT CREATED A DEFEAT ON PAGE 15.

*First case:* Two motions in one and the same line and direction belong to one and the same point.

Two speeds AB and ab are to be represented as contained in one speed of motion. Let these speeds be assumed to be equal for the moment, so that  $AB = ab$ ; then I say that they can't be represented at the same time in one and the same space (whether absolute or relative) in one and the same point. For, since the lines AB and ab designating the speeds are, properly speaking, the spaces they traverse in equal times, the composition of these spaces  $AB$  and  $ab = BC$ , and hence the line AC as the sum of the spaces, would have to express the sum of the two speeds. But neither the part AB nor the part BC represents the speed = ab, for they are not traversed in the same time as ab. Therefore, the doubled line AC, traversed in the same time as the line ab, does not represent the twofold speed of the latter, which, however, was required. Therefore, the composition of two speeds in one direction cannot be represented intuitively in the same space.

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·THE PASSAGE THAT CREATED A DEFEAT ON PAGE 51, FIRST IN ELLINGTON'S TRANSLATION:·

Now, the proper motion of matter is a predicate which determines such motion's subject (the movable) and with regard to matter as a multitude of movable parts indicates the plurality of the moved subjects (at equal velocity in the same direction); this is not the case with dynamical properties, whose quantity can also be the quantity of the action of a single subject (e.g., a particle of air can have more or less elasticity). Because of all of this it is clear that the quantity of substance in a matter must be estimated mechanically, i.e., by the quantity of the proper motion of the matter, and not dynamically, by the quantity of its original moving forces. Nevertheless, original attraction as the cause of universal gravitation can indeed provide a measure of the quantity of matter and its substance (as actually happens in the comparison of matters by weighing), although there seems to be laid at the foundation here not the proper motion of the attracting matter but a dynamical measure, namely, attractive force. But in the case of this force, the action of one matter occurs with all its parts directly on all parts of another matter; and hence the action is (at equal distances) obviously proportional to the number of the parts. Because of this fact the attracting body itself thereby also imparts the velocity of its proper motion (by means of the resistance of the attracted body). This velocity is directly proportional, in equivalent external circumstances, to the number of the attracting body's parts; because of this the estimation takes place here, as a matter of fact, mechanically, although only indirectly so.

·AND THEN IN FRIEDMAN'S:·

Now since the *inherent motion* of matter is a predicate that determines its subject (the movable), and indicates in a matter, as an aggregate of movables, a plurality of the subjects moved (at the same speed and in the same way), which is not the case for dynamical properties, whose magnitude can also be that of the action of a single subject (where an air particle, for example, can have more or less elasticity); it therefore becomes clear how the quantity of substance in a matter has to be estimated mechanically only, that is, by the quantity of its own inherent motion, and not dynamically, by that of the original moving forces. Nevertheless, *original attraction*, as the cause of universal gravitation, can still yield a measure of the quantity of matter, and of its substance (as actually happens in the comparison of matters by weighing), even though a dynamical measure—namely, attractive force—seems here to be the basis, rather than the attracting matter's own inherent motion. But since, in the case of this force, the action of a matter with all its parts is exerted immediately on all parts of another, and hence (at equal distances) is obviously proportional to the aggregate of the parts, the attracting body also thereby imparts to itself a speed of its own inherent motion (by the resistance of the attracted body), which, in like external circumstances, is exactly proportional to the aggregate of its parts; so the estimation here is still in fact mechanical, although only indirectly so.